ON LEFT IDEALS OF PRIME RINGS
WITH GENERALIZED DERIVATIONS

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Abstract

In this paper the author considers a prime ring $R$ with characteristic different from two and extends some well known results concerning derivations of prime rings to the generalized derivation $f: R \to R$ associated with a derivation $d$ of $R$ and a nonzero left ideal $U$ of $R$ which is semiprime as a ring.

Keywords: Prime ring, Derivation, Generalized derivation, Homomorphism, Anti-homomorphism.

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1. Introduction

Throughout this paper, $R$ will be a prime ring with characteristic different from two and $I$ a nonzero left ideal of $R$ which is semiprime as a ring, $Z$ the multiplicative center of $R$, $Q_r(R)$ the right Martindale ring of quotients, $C$ the extended centroid and $RC = RC$ the central closure. For any $x, y \in R$, the symbol $[x, y]$ will represent the commutator $xy - yx$. An additive mapping $f: R \to R$ is called a generalized derivation if there exists a derivation $d: R \to R$ such that

$$f(xy) = f(x)y + xd(y)$$

for all $x, y \in R$. The concept of generalized derivation includes the concept of derivation. Moreover, a generalized derivation with $d = 0$ includes the concept of left multiplier, that is an additive map satisfying $f(xy) = f(x)y$, for all $x, y \in R$.

The study of the commutativity of prime rings with derivations was initiated by E. C. Posner [10]. Over the last two decades, a lot of work has been done on this subject. Recently, M. Bresar defined a generalized derivation in [5]. Many authors have investigated the properties of prime or semiprime rings with generalized derivations. In the present paper our objective is to generalize some results obtained in [2], [3], [4], [7] and [9] for generalized derivations and a left ideal of a prime ring $R$ which is semiprime as a ring.

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