

INEQUALITIES FOR ONE SIDED APPROXIMATION IN ORLICZ SPACES

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Abstract

In the present article some inequalities of trigonometric approximation are proved in Orlicz spaces generated by a quasiconvex Young function. Also, the main one-sided approximation problems are investigated.

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1. Introduction

A function Φ is called a *Young function* if Φ is even, continuous, nonnegative in $\mathbb{R} := (-\infty, +\infty)$, increasing on $\mathbb{R}^+ := (0, \infty)$ and such that

$$\Phi(0) = 0, \lim_{x \rightarrow \infty} \Phi(x) = \infty.$$

A function $\varphi : [0, \infty) \rightarrow [0, \infty)$ is said to be *quasiconvex* if there exist a convex Young function Φ and a constant $c_1 \geq 1$ such that

$$\Phi(x) \leq \varphi(x) \leq \Phi(c_1 x) \quad \forall x \geq 0.$$

Set $\mathbb{T} := [0, 2\pi]$ and let φ be a quasiconvex Young function. We denote by $\varphi(L)$ the class of complex valued Lebesgue measurable functions $f : \mathbb{T} \rightarrow \mathbb{C}$ satisfying the condition

$$\int_{\mathbb{T}} \varphi(|f(x)|) dx < \infty.$$

The class of functions $f : \mathbb{T} \rightarrow \mathbb{C}$ having the property

$$\int_{\mathbb{T}} \varphi(c_2 |f(x)|) dx < \infty$$

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