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STRONGLY EXTENDING MODULES

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Abstract

In this paper, we recall the concept of strong largeness to define strongly extending modules which are particular extending modules, and investigate some properties of strongly extending modules. We supply some examples showing that extending modules need not be strongly extending. Under some conditions we prove that extending modules are strongly extending.

Keywords: Large submodules, extending modules, strongly large submodules, strongly extending modules.

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1. Introduction

Throughout all rings have identities and all modules are unital right modules. Let R be a ring and M an R-module. For submodules A and B of M, $A \leq B$ denotes A is a submodule of B and $S = \mathrm{End}_R(M)$ denotes the ring of right R-module endomorphisms of M. Then M is a left S-module, right R-module and (S, R)-bimodule. In this work, for any rings S and R and any (S, R)-bimodule M, $r_R(.)$ and $l_M(.)$ denote the right annihilator of a subset of M in R and the left annihilator of a subset of R in M, respectively. Similarly, $l_S(.)$ and $r_M(.)$ are the left annihilator of a subset of M in S and the right annihilator of a subset of S in M, respectively. For $m \in M$ and $N \leq M$, the right ideal $\{r \in R \mid mr \in N\}$ of R is denoted by $m^{-1}N$. When $N = 0$ the right ideal $m^{-1}N$ and $r_R(m)$ coincide. It is clear that N is a large submodule of M if and only if $m(m^{-1}N) \neq 0$ for each nonzero $m \in M$.

In this paper, our aim is to introduce and study strongly extending modules by using the concept of strong largeness. We see that some known essential objects are in fact strongly large and clearly every nonzero ideal in a commutative domain enjoys this property. We make use strong large submodules to define strongly extending modules. At first we give some elementary properties of strongly large submodules and introduce strongly

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This paper is dedicated to Professor Abdullah Harmanci on his 70th birthday.