

## RESULTS ON THE SUPREMUM OF FRACTIONAL BROWNIAN MOTION\*

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### Abstract

We show that the distribution of the square of the supremum of reflected fractional Brownian motion up to time  $a$ , with Hurst parameter- $H$  greater than  $1/2$ , is related to the distribution of its hitting time to level 1, using the self similarity property of fractional Brownian motion. It is also proven that the second moment of supremum of reflected fractional Brownian motion up to time  $a$  is bounded above by  $a^{2H}$ . Similar relations are obtained for the supremum of fractional Brownian motion with Hurst parameter greater than  $1/2$ , and its hitting time to level 1. What is more, we obtain an upper bound on the complementary probability distribution of the supremum of fractional Brownian motion and reflected fractional Brownian motion up to time  $a$ , using Jensen's and Markov's inequalities. A sharper bound is observed on the distribution of the supremum of fractional Brownian motion by the properties of Gamma distribution. Finally, applications of the given results to financial markets are investigated, and partial results are provided.

**Keywords:** Fractional Brownian motion, Reflected fractional Brownian motion, Self similarity property, Hitting time, Gamma distribution, Hurst parameter, Markov's inequality, Jensen's inequality.

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