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ON WEAK SYMMETRIES OF ALMOST KENMOTSU (κ, μ, ν)-SPACES

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Abstract

In this paper, we study on weak symmetries of almost Kenmotsu (κ, μ, ν)-spaces. For each α, γ, δ 1-forms and any vector field X we get $\alpha + \gamma + \delta = \frac{X(\kappa)}{\kappa}$, if a $(2n+1)$-dimensional almost Kenmotsu (κ, μ, ν)-space is weakly symmetric and for each ε, σ, ρ 1-forms we get $\varepsilon + \sigma + \rho = \frac{X(\kappa)}{\kappa}$, if a $(2n+1)$-dimensional almost Kenmotsu (κ, μ, ν)-space is weakly Ricci symmetric.

Keywords: Almost Kenmotsu Manifolds, Weak Symmetric Manifolds, Almost Kenmotsu (κ, μ, ν)-Spaces.

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1. INTRODUCTION

Weakly symmetric Riemannian manifolds are generalizations of locally symmetric manifolds and pseudo-symmetric manifolds. These are manifolds in which the covariant derivative DR of the curvature tensor R is a linear expression in R. The appearing coefficients of this expression are called associated 1-forms. They satisfy in the specified types of manifolds gradually weaker conditions.

Firstly, the notions of weakly symmetric and weakly Ricci-symmetric manifolds were introduced by L. Tamassy and T. Q. Binh in 1992 ([10] and [11]). In [10], the authors considered weakly symmetric and weakly projective-symmetric Riemannian manifolds. In 1993, the authors considered weakly symmetric and weakly Ricci-symmetric Einstein and Sasakian manifolds [11]. In 2000, U. C. De, et. all gave necessary conditions for

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the compatibility of several K-contact structures with weak symmetry and weakly Ricci-symmetry [13]. In 2002, C. Özgür, considered weakly symmetric and weakly Ricci-symmetric Lorentzian para-Sasakian manifolds [14]. Recently in [8], C. Özgür studied weakly symmetric Kenmotsu manifolds and in [15] Aktan and Görgülü studied on weak symmetries of almost r-paracontact Riemannian manifold of P-Sasakian type.

Manifolds known as Kenmotsu manifolds have been introduced and studied by K. Kenmotsu in 1972 [16]. Kenmotsu manifolds were studied by many authors such as [1]-[9] and many others. T. Koufogiorgos, et. all, introduced in [17] the notion of (κ,μ,ν)-contact metric manifold, where now the equation to be satisfied is

$$R(X,Y)\xi = \eta(Y)(\kappa I + \mu h + \nu\varphi h)X - \eta(X)(\kappa I + \mu h + \nu\varphi h)Y, \tag{1.1}$$

for some smooth functions κ,μ and ν on M. Lastly, H. Öztürk, N. Aktan and C. Murathan studied in [18] the almost α-cosymplectic (κ, μ, ν)-spaces under different conditions (like η-parallelism) and gave an interesting example in dimension 3. Some of their results are used in this paper.

These almost Kenmotsu manifolds whose almost Kenmotsu structures (φ, ξ, η, g) satisfy the condition

$$\begin{aligned} R(\xi, X)Y = \; & \kappa(g(Y,X)\xi - \eta(X)Y) + \mu(g(hY,X)\xi - \eta(Y)hX) \\ & + \nu(g(\varphi hY, X)\xi - \eta(Y)\varphi hX), \end{aligned} \tag{1.2}$$

for $\kappa, \mu, \nu \in \Re_n(M^{2n+1})$, where $\Re_n(M^{2n+1})$ be the subring of the ring of smooth functions f on M^{2n+1} for which $df \wedge \eta = 0$.

A non-flat differentiable manifold M^{2n+1} is called weakly symmetric if there exist a vector field P and 1-forms $\alpha, \beta, \gamma, \delta$, on M such that

$$\begin{aligned} (\nabla_X R)(Y,Z,W) = \; & \alpha(X)R(Y,Z)W + \beta(Y)R(X,Z)W \\ & + \gamma(Z)R(Y,X)W + \delta(W)R(Y,Z)X + g(R(Y,Z)W,X)P, \end{aligned} \tag{1.3}$$

holds for all vector fields $X, Y, Z, W \in \chi(M^{2n+1})$ ([10] and [11]). A weakly symmetric manifold (M^{2n+1}, g) is pseudo-symmetric if $\beta = \gamma = \delta = \frac{1}{2}\alpha$ and $P = A$, locally symmetric if $\alpha = \beta = \gamma = \delta = 0$ and $P = 0$. A weakly symmetric is said to be proper if at least one of the 1-forms $\alpha, \beta, \gamma, \delta$ is not zero or $P \neq 0$.

A differentiable manifold M^{2n+1} is called weakly Ricci-symmetric if there exists 1-forms $\varepsilon, \sigma, \rho$ such that the condition

$$(\nabla_X S)(Y,Z) = \varepsilon(X)S(Y,Z) + \sigma(Y)S(X,Z) + \rho(Z)S(X,Y), \tag{1.4}$$

holds for all vector fields $X, Y, Z, W \in \chi(M^{2n+1})$ ([10] and [11]). If $\varepsilon = \sigma = \rho$ then M^{2n+1} is called pseudo Ricci-symmetric [12].

From (1.4), an easy calculation shows that if M^{2n+1} is weakly symmetric then we have

$$\begin{aligned} (\nabla_X S)(Z,W) = \; & \alpha(X)S(Z,W) + \beta(R(X,Z)W) \\ & + \gamma(Z)S(X,W) + \delta(W)S(X,Z) + \rho(R(X,W)Z), \end{aligned} \tag{1.5}$$

where P is defined by $\rho(X) = g(X,P)$ for all $X \in \chi(M^{2n+1})$ [11].

In this paper, we consider weakly symmetries and weakly Ricci symmetries for almost Kenmotsu (κ, μ, ν)-spaces.

2. ALMOST KENMOTSU (κ,μ,ν)-SPACES

Let $\left(M^{2n+1},\varphi,\xi,\eta,g\right)$ be a $(2n+1)$-dimensional almost contact Riemannian manifold, where φ is a $(1,1)$-tensor field, ξ is the structure vector field, η is a 1-form and g is Riemannian metric. It is well-known that φ,ξ,η,g satisfy

$$\eta(\xi)=1,\quad \varphi\xi=0,\quad \eta\circ\varphi=0, \tag{2.1}$$

$$\varphi^2X=-X+\eta(X)\,\xi,\quad \eta(X)=g(X,\xi), \tag{2.2}$$

$$g(\varphi X,\varphi Y)=g(X,Y)-\eta(X)\,\eta(Y), \tag{2.3}$$

for any vector fields X,Y on M^{2n+1}.

The 2-form Φ of M^{2n+1} defined by $\Phi(X,Y)=g(\phi X,Y)$, is called the fundamental 2-form of the almost contact metric manifold M^{2n+1}. Almost contact metric manifolds such that $d\eta=0$ and $d\Phi=2\eta\wedge\Phi$ are almost Kenmotsu manifolds. Finally, a normal almost Kenmotsu manifold is called Kenmotsu manifold. An almost Kenmotsu manifold is a nice example of an almost contact manifold which is not K-contact (and hence not a Sasakian) manifold.

Now let us recall some important curvature-properties of almost Kenmotsu manifolds satisfy (1.1) and (1.2) and the following properties:

$$(\nabla_{X\varphi})\,Y=g(\varphi X+hX,Y)\,\xi-\eta(Y)\,(\varphi X+hX), \tag{2.4}$$

$$\nabla_X\xi=-\varphi^2X-\varphi hX, \tag{2.5}$$

$$S(X,\xi)=2n\kappa\eta(X), \tag{2.6}$$

$$Q\xi=2n\kappa\xi, \tag{2.7}$$

$$l=-\kappa\varphi^2+\mu h+\nu\varphi h, \tag{2.8}$$

$$l\varphi-\varphi l=2\mu h\varphi+2\nu h, \tag{2.9}$$

$$h^2=\left(\kappa^2+1\right)\varphi^2,\text{for }\kappa\leq-1, \tag{2.10}$$

$$(\nabla_\xi h)=-\mu\varphi h+(v-2)\,h. \tag{2.11}$$

Here, l and Jacobi operator are defined by $l(X)=R(X,\xi)\,\xi$ and $h=\dfrac{1}{2}L_\xi\varphi$, where L is Lie derivative operator.

2.1. Theorem. *On almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, the functions κ, μ, ν only vary in the direction of ξ, i.e.*

$$X(\kappa)=X(\mu)=X(\nu)=0$$

for every vector field X orthogonal to ξ [18].

3. MAIN RESULTS

3.1. Theorem. *Let M be an almost Kenmotsu (κ,μ,ν)-space. If M is weakly symmetric then* $\alpha+\gamma+\delta=\dfrac{X(\kappa)}{\kappa}$.

Proof. Assume that M^{2n+1} is a weakly symmetric almost Kenmotsu (κ,μ,ν)-space. Putting $W=\xi$ in (1.5) we get

$$\begin{aligned}(\nabla_X S)(Z,\xi) = \;& \alpha(X)S(Z,\xi)+\beta(R(X,Z)\xi)\\ &+\gamma(Z)S(X,\xi)+\delta(\xi)S(X,Z)+\rho(R(X,\xi)Z),\end{aligned} \tag{3.1}$$

So, using (1.1) (1.2) and (2.6) we have

$$\begin{aligned}&(\nabla_X S)(Z,\xi)=\\ &2n\kappa\alpha(X)+\beta(\kappa)\eta(Z)X+\kappa\eta(Z)\beta(X)\\ &+\beta(\mu)\eta(Z)hX+\mu\eta(Z)\beta(hX)+\beta(\nu)\eta(Z)\varphi hX\\ &+\nu\eta(Z)\beta(\varphi hX)-\beta(\kappa)\eta(X)Z-\kappa\eta(X)\beta(Z)\\ &-\beta(\mu)\eta(X)hZ-\mu\eta(X)\beta(hZ)-\beta(\nu)\eta(X)\varphi hZ\\ &-\nu\eta(X)\beta(\varphi hZ)+2n\kappa\gamma(Z)\eta(X)+\delta(\xi)S(X,Z)\\ &-\rho(\kappa)(g(X,Z)\xi-\eta(Z)X)-\kappa(g(X,Z)\rho(\xi)-\eta(Z)\rho(X))\\ &-\rho(\mu)(g(hZ,X)\xi-\eta(Z)hX)-\mu(g(hZ,X)\rho(\xi)-\eta(Z)\rho(hX))\\ &-\rho(\nu)(g(\varphi hZ,X)\xi-\eta(Z)\varphi hX)\\ &-\nu(g(\varphi hZ,X)\rho(\xi)-\eta(Z)\rho(\varphi hX)).\end{aligned} \tag{3.2}$$

By the covariant differentiation of the Ricci tensor S, the left side can be written as

$$(\nabla_X S)(Z,\xi)=\nabla_X S(Z,\xi)-S(\nabla_X Z,\xi)-S(Z,\nabla_X\xi).$$

By the use of (2.5), (2.6)and the parallelity of the metric tensor g we have

$$(\nabla_X S)(Z,\xi)=2nX(\kappa)\eta(Z)+2n\kappa g(Z,\nabla_X\xi)-S(Z,\nabla_X\xi). \tag{3.3}$$

Comparing the right hand sides of (3.2) and (3.3), we obtain

$$\begin{aligned}&2nX(\kappa)\eta(Z)+2n\kappa g(Z,\nabla_X\xi)-S(Z,\nabla_X\xi)=\\ &2n\kappa\alpha(X)+\beta(\kappa)\eta(Z)X+\kappa\eta(Z)\beta(X)\\ &+\beta(\mu)\eta(Z)hX+\mu\eta(Z)\beta(hX)\\ &+\beta(\nu)\eta(Z)\varphi hX+\nu\eta(Z)\beta(\varphi hX)\\ &-\beta(\kappa)\eta(X)Z-\kappa\eta(X)\beta(Z)\\ &-\beta(\mu)\eta(X)hZ-\mu\eta(X)\beta(hZ)-\beta(\nu)\eta(X)\varphi hZ\\ &-\nu\eta(X)\beta(\varphi hZ)+2n\kappa\gamma(Z)\eta(X)\\ &+\delta(\xi)S(X,Z)-\rho(\kappa)(g(X,Z)\xi-\eta(Z)X)\\ &-\kappa(g(X,Z)\rho(\xi)-\eta(Z)\rho(X))\\ &-\rho(\mu)(g(hZ,X)\xi-\eta(Z)hX)\\ &-\mu(g(hZ,X)\rho(\xi)-\eta(Z)\rho(hX))\\ &-\rho(\nu)(g(\varphi hZ,X)\xi-\eta(Z)\varphi hX)\\ &-\nu(g(\varphi hZ,X)\rho(\xi)-\eta(Z)\rho(\varphi hX)).\end{aligned} \tag{3.4}$$

Putting $X=Z=\xi$ in (3.4) and using (2.1), (2.2), (2.3) and (2.6) we get

$$2n\kappa[\alpha(\xi)+\gamma(\xi)+\delta(\xi)]=2n\xi(\kappa).$$

Since $n>1$ and $\kappa\neq 0$, we obtain

$$\alpha(\xi)+\gamma(\xi)+\delta(\xi)=\frac{\xi(\kappa)}{\kappa}. \tag{3.5}$$

So, vanishing of the 1-form $\alpha+\gamma+\delta$ over the vector field ξ necessary in order that M^{2n+1} be an almost Kenmotsu (κ,μ,ν)-space. Now we will show that

$$\alpha(X)+\gamma(X)+\delta(X)=\frac{X(\kappa)}{\kappa}$$

holds for all vector fields on M^{2n+1}.

In (1.5) $Z=\xi$, similar to the previous calculations it follows that

$$\begin{aligned}
&2nX(\kappa)\eta(W)+2n\kappa g(W,\nabla_X\xi)-S(\nabla_X\xi,W)=\\
&2n\kappa\alpha(X)\eta(W)+\beta(\kappa)(g(X,W)\xi-\eta(W)X)\\
&-\kappa(g(X,W)\beta(\xi)-\eta(W)\beta(X))\\
&-\beta(\mu)(g(hW,X)\xi-\eta(W)hX)\\
&-\mu(g(hW,X)\beta(\xi)-\eta(W)\beta(hX))\\
&-\beta(\nu)(g(\varphi hW,X)\xi-\eta(W)\varphi hX)\\
&-\nu(g(\varphi hW,X)\beta(\xi)-\eta(W)\beta(\varphi hX))+\gamma(\xi)S(X,W)\\
&+2n\kappa\delta(W)\eta(X)+\rho(\kappa)\eta(W)X\\
&+\kappa\eta(W)\rho(X)+\rho(\mu)\eta(W)hX\\
&+\mu\eta(W)\rho(hX)+\rho(\nu)\eta(W)\varphi hX\\
&+\nu\eta(W)\rho(\varphi hX)-\rho(\kappa)\eta(X)W\\
&-\kappa\eta(X)\rho(W)-\rho(\mu)\eta(X)hW\\
&-\mu\eta(X)\rho(hW)-\rho(\nu)\eta(X)\varphi hW-\nu\eta(X)\rho(\varphi hW).
\end{aligned}\tag{3.6}$$

Replacing W with ξ in (3.6) and by making use of (1.1) 2.1), (2.2), (2.3) and (2.6) we have

$$\begin{aligned}
2nX(\kappa)=\;&2n\kappa\alpha(X)-\beta(\kappa)(\eta(X)\xi-X)\\
&-\kappa(\eta(X)\beta(\xi)-\beta(X))\\
&+\beta(\mu)hX+\mu\beta(hX)\\
&+\beta(\nu)\varphi hX+\nu\beta(\varphi hX)+2n\kappa\gamma(\xi)\eta(X)\\
&+2n\kappa\delta(\xi)\eta(X)+\rho(\kappa)\nu+\kappa\rho(X)\\
&+\rho(\mu)hX+\mu\rho(hX)+\rho(\nu)\varphi hX\\
&+\nu\rho(\varphi hX)-\rho(\kappa)\eta(X)\xi-\kappa\eta(X)\rho(\xi).
\end{aligned}\tag{3.7}$$

Taking $X=\xi$ in (3.6) and replacing W with X we get

$$\begin{aligned}
2n\xi(\kappa)\eta(X)=\;&2n\kappa\alpha(\xi)\eta(X)+2n\kappa\gamma(\xi)\eta(X)+2n\kappa\delta(X)\\
&+\rho(\kappa)\eta(X)\xi+\kappa\rho(\xi)\eta(X)-\rho(\kappa)X-\kappa\rho(X)\\
&-\rho(\mu)hX-\mu\rho(hX)-\rho(\nu)\varphi hX-\nu\rho(\varphi hX).
\end{aligned}\tag{3.8}$$

Now putting $X=\xi$ in (3.4) and replacing Z with X we have

$$\begin{aligned}
2n\xi(\kappa)\eta(X)=\;&2n\kappa\alpha(\xi)\eta(X)+\beta(\kappa)\eta(X)\xi+\kappa\eta(X)\beta(\xi)\\
&-\beta(\kappa)X-\kappa\beta(X)-\beta(\mu)hX-\mu\beta(hX)-\beta(\nu)\varphi hX\\
&-\nu\beta(\varphi hX)+2n\kappa\gamma(X)+2n\kappa\eta(X)\delta(\xi)
\end{aligned}\tag{3.9}$$

Taking (3.7), (3.8) and (3.9) we get

$$2nX(\kappa)=2n\kappa\left[\alpha(X)+\gamma(X)+\delta(X)\right],$$

for all X. This implies

$$\alpha(X)+\gamma(X)+\delta(X)=\frac{X(\kappa)}{\kappa},$$

which completes the proof of the theorem. □

3.2. Theorem. *Let M be an almost Kenmotsu (κ,μ,ν)-space. If M is weakly Ricci symmetric then* $\varepsilon+\sigma+\rho=\dfrac{X(\kappa)}{\kappa}$.

Proof. Assume that M^{2n+1} is a weakly Ricci symmetric almost Kenmotsu (κ,μ,ν)-space. Putting $Z=\xi$ in (1.4) and using (2.6) we have

$$(\nabla_X S)(Y,\xi)=2n\kappa\varepsilon(X)\eta(Y)+2n\kappa\sigma(Y)\eta(X)+\rho(\xi)S(X,Y). \tag{3.10}$$

Replacing Z with Y in (3.3) and comparing the right hand sides of the equations (3.10) and (3.3) we obtain

$$\begin{aligned}2nX(\kappa)\eta(Y)+2n\kappa g(Y,\nabla_X\xi)-S(Y,\nabla_X\xi)= &\ 2n\kappa\varepsilon(X)\eta(Y)+2n\kappa\sigma(Y)\eta(X)\\ &+\rho(\xi)S(X,Y).\end{aligned} \tag{3.11}$$

Taking $X=Y=\xi$ in (3.11) by making use of (2.1), (2.2), (2.3) and (2.6) we get

$$2n\xi(\kappa)=2n\kappa\left[\varepsilon(\xi)+\sigma(\xi)+\rho(\xi)\right],$$

which gives, (since $n>1$ and $\kappa\neq 0$),

$$\varepsilon(\xi)+\sigma(\xi)+\rho(\xi)=\frac{\xi(\kappa)}{\kappa}. \tag{3.12}$$

Putting $X=\xi$ in (3.11) we have

$$2n\xi(\kappa)\eta(Y)=2n\kappa\eta(Y)\left[\varepsilon(\xi)+\rho(\xi)\right]+2n\kappa\sigma(Y).$$

So by virtue of (3.12) this yields

$$\sigma(Y)=\sigma(\xi)\eta(Y).$$

Replacing Y with X we get

$$\sigma(X)=\sigma(\xi)\eta(X). \tag{3.13}$$

Similarly taking $Y=\xi$ in (3.11) we also have

$$\varepsilon(X)=\varepsilon(\xi)\eta(X). \tag{3.14}$$

Since $(\nabla_\xi S)(\xi,X)=0$,we obtain

$$\rho(X)=\rho(\xi)\eta(X). \tag{3.15}$$

Therefore the summation of the equations (3.13), (3.14) and (3.15) give us

$$\varepsilon(X)+\sigma(X)+\rho(X)=\frac{X(\kappa)}{\kappa}$$

for all X. Our theorem is proved. □

By virtue of the above theorems we have the following corollaries:

3.3. Corollary. *Let M be an almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, and the functions $\kappa,\ \mu\ \nu$ only vary in the direction of ξ. There exists no weakly symmetric almost Kenmotsu (κ,μ,ν)-space $M^{2n+1},(\kappa\leq -1)$, if $\alpha+\gamma+\delta$ is not everywhere zero.*

3.4. Corollary. *Let M be an almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, and the functions $\kappa,\ \mu\ \nu$ only vary in the direction of ξ. There exists no weakly Ricci-symmetric almost Kenmotsu (κ,μ,ν)-space $M^{2n+1},(\kappa\leq -1)$, if $\varepsilon+\sigma+\rho$ is not everywhere zero.*

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