ON OPERATORS OF STRONG TYPE B

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Received 17:06:2011 : Accepted 21:03:2012

Abstract
We discuss operators of strong type B between a Banach lattice and a Banach space and give necessary and sufficient conditions for this class of operators to coincide with weakly compact operators.

Keywords: Operators of strong type B, b-weakly compact operators, Banach lattice.

2000 AMS Classification: 46 A 40, 46 B 40, 46 B 42.

1. Introduction
A vector lattice \( E \) is an ordered vector space for which \( \text{sup}\{x, y\} \) exists for every pair of vectors \( x, y \) in \( E \). Let \( E \) be a vector lattice. For \( x, y \in E \) with \( x \leq y \) in \( E \), the set \( [x, y] = \{ t \in E : x \leq t \leq y \} \) is called an order interval. A subset of \( E \) is called order bounded if it is contained in some order interval. A Banach lattice \( E \) is a Banach space \((E, || \cdot ||)\) where \( E \) is also a vector lattice and its norm satisfies the following property: For each \( x, y \in E \) with \( |x| \leq |y| \), we have \( ||x|| \leq ||y|| \). If \( E \) is a Banach lattice, its topological dual \( E' \) equipped with the dual norm and order is also a Banach lattice. A norm \( || \cdot || \) on a Banach lattice \( E \) is called order continuous if for each net \((x_\alpha)\) with \( x_\alpha \downarrow 0 \) in \( E \), \((x_\alpha) \) converges to zero for the norm \( || \cdot || \), where \((x_\alpha) \downarrow 0 \) means that \((x_\alpha) \) is decreasing, its infimum exists and is equal to zero.

A Banach lattice \( E \) is said to be a KB-space whenever every increasing norm bounded sequence in \( E_+ = \{ x \in E : 0 \leq x \} \) is norm convergent. Each KB-space has order continuous norm, but a Banach lattice with an order continuous norm is not necessarily a KB-space. Indeed, the Banach lattice \( c_0 \) has order continuous norm but it is not a KB-space. However, if \( E \) is a Banach lattice, the topological dual is a KB-space if and only if its norm is order continuous. A Banach lattice \( E \) is an abstract M-space (AM-space in short) if for each \( x, y \in E \) with \( \inf\{x, y\} = 0 \), we have \( ||x + y|| = \max\{||x||, ||y||\} \). A Banach lattice \( E \) is an AL-space if its dual \( E' \) is an AM-space.

We will use the term operator to mean a bounded linear mapping. The space of bounded linear operators between Banach spaces \( E, F \) will be denoted by \( L(E, F) \). All vector lattices considered in this note are assumed to have separating order duals. We refer the reader to [1] and [18] for further terminology and notation.

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