

# SUMS OF PRODUCTS OF THE TERMS OF THE GENERALIZED LUCAS SEQUENCE $\{V_{kn}\}$

Emrah Kılıç\*, Yücel Türker Ulutaş<sup>†</sup> and Neşe Ömür<sup>†‡</sup>

Received 09:06:2010 : Accepted 11:10:2010

## Abstract

In this study we consider the generalized Lucas sequence  $\{V_n\}$  with indices in arithmetic progression. We also compute the sums of products of the terms of the Lucas sequence  $\{V_{kn}\}$  for positive odd integers  $k$ .

**Keywords:** Second order linear recurrence, Fibonomial coefficients.

*2010 AMS Classification:* 11 B 37, 15 A 23, 11 B 39, 11 C 20.

Communicated by Yücel Tıraş

## 1. Introduction

The binary linear recurrence  $W_n = W_n(a, b; p, q)$  is defined as follows for  $n > 1$ ,

$$W_n = pW_{n-1} + qW_{n-2},$$

where  $W_0 = a, W_1 = b$ .

The Binet formula for  $\{W_n\}$  is

$$(1.1) \quad W_n = A\alpha^n + B\beta^n,$$

where  $A = \frac{b-a\beta}{\alpha-\beta}$ ,  $B = \frac{a\alpha-b}{\alpha-\beta}$  and  $\alpha, \beta = \left(p \pm \sqrt{p^2 + 4q}\right) / 2$ .

For  $n > 1$  and a fixed positive integer  $k$ , the terms of  $\{W_{kn}\}$  satisfy the recursion [6, 7]:

$$W_{kn} = V_k W_{k(n-1)} - (-q)^k W_{k(n-2)},$$

---

\*TOBB University of Economics and Technology, Mathematics Department, 06560 Ankara, Turkey. E-mail: [ekilic@etu.edu.tr](mailto:ekilic@etu.edu.tr)

<sup>†</sup>Kocaeli University, Mathematics Department, 41380 İzmit, Kocaeli, Turkey.  
E-mail: (Y. T. Ulutaş) [turkery@kocaeli.edu.tr](mailto:turkery@kocaeli.edu.tr) (N. Ömür) [neseomur@kocaeli.edu.tr](mailto:neseomur@kocaeli.edu.tr)

<sup>‡</sup>Corresponding Author.