

DYNAMICS AND GLOBAL BEHAVIOR FOR A FOURTH-ORDER RATIONAL DIFFERENCE EQUATION

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Abstract

In this paper we study the behavior of the rational difference equation of the fourth order

$$x_{n+1} = ax_n + \frac{bx_n x_{n-2}}{cx_{n-2} + dx_{n-3}}, \quad n = 0, 1, \dots,$$

where the initial conditions $x_{-3}, x_{-2}, x_{-1}, x_0$ are arbitrary positive real numbers and a, b, c, d are positive constants. Also, we give the solution of some special cases of this equation.

Keywords: difference equations, stability, boundedness, periodicity, solution of difference equations.

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1. Introduction

In this paper we deal with the behavior of the solutions of the following nonlinear difference equation

$$(1) \quad x_{n+1} = ax_n + \frac{bx_n x_{n-2}}{cx_{n-2} + dx_{n-3}}, \quad n = 0, 1, \dots,$$

where the initial conditions $x_{-3}, x_{-2}, x_{-1}, x_0$ are arbitrary positive real numbers and a, b, c, d are positive constants. Also, we give the solution of some special cases of this equation.

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