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AN UPPER BOUND ON THE SPECTRAL RADIUS OF WEIGHTED GRAPHS

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Abstract

We consider weighted graphs, where the edge weights are positive definite matrices. The eigenvalues of a graph are the eigenvalues of its adjacency matrix. We obtain another upper bound which is sharp on the spectral radius of the adjacency matrix and compare with some known upper bounds with the help of some examples of graphs. We also characterize graphs for which the bound is attained.

Keywords: Weighted graph, Adjacency matrix, Spectral radius, Upper bound

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1. Introduction

We consider simple graphs, that is, graph which have no loops or parallel edges. Hence a graph $G = (V, E)$ consists of a finite set of vertices, V, and a set of edges, E, each of whose elements are an unordered pair of distinct vertices. Generally, V is taken as $V = \{1, 2, .., n\}$.

A weighted graph is a graph, each edge of which has been assigned a square matrix, called the weight of the edge. All the weight matrices will be assumed to be of same order and will be assumed to be positive matrix. In this paper, by "weighted graph" we will mean "a weighted graph with each of its edges bearing a positive definite matrix as weight", unless otherwise stated.

Now we introduce some notations. Let G be a weighted graph on n vertices. Denote by $w_{i,j}$ the positive definite weight matrix of order p of the edge ij, and assume that $w_{i,j} = w_{j,i}$ .We write $i \sim j$ if vertices i and j are adjacent. Let $w_i = \sum_{j:j\sim i} w_{i,j}$.

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