MODULES WHOSE MAXIMAL SUBMODULES ARE SUPPLEMENTS

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Abstract

We study modules whose maximal submodules are supplements (direct summands). For a locally projective module, we prove that every maximal submodule is a direct summand if and only if it is semisimple and projective. We give a complete characterization of the modules whose maximal submodules are supplements over Dedekind domains.

Keywords: Locally projective module, Supplement submodule.


1. Introduction

Let \( R \) be a unitary ring and \( M \) a left \( R \)-module. A submodule \( N \) of \( M \) is called a supplement if there exists another submodule \( L \) such that \( N \) is minimal with respect to the property that \( N + L = M \). This is equivalent to \( N + L = M \) and \( N \cap L \ll N \). A module \( M \) is called supplemented if every submodule has a supplement. Several authors have been recently attracted by different generalizations of supplemented modules. An interesting example of this situation has been studied in [1], where modules \( M \) in which the kernel of any epimorphism from \( M \) to a finitely generated module has a supplement are studied. These modules are characterized as modules whose maximal submodules have supplements, (see, [1, Theorem 2.8]). Motivated by these results, we study in this paper, modules in which every maximal submodule is a supplement, and modules in which every maximal submodule is a direct summand. For the sake of brevity, we call them ms-modules and md-modules, respectively.

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