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COMMON FIXED POINT RESULT IN ORDERED CONE METRIC SPACES

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Abstract

Fixed point and common fixed point results for generlized contractive mappings are obtianed in ordered cone metric spaces.

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1. Introduction and Preliminaries

Recently, Huang and Zhang [4] introduced the concept of a cone metric space, replacing the set of positive real numbers by an ordered Banach space. They obtained some fixed point theorems in cone metric spaces using the normality of cone which induces an order in Banach spaces. Rezapour and Hamlbarani [9] showed the existence of a non normal cone metric space and obtained some fixed point results in cone metric spaces. Subsequently, Abbas and Rhoades [1] studied common fixed point theorems in cone metric spaces (see also, [5, 7, 8]). Recently Altun et al. [2] proved some fixed point and common fixed point theorems in ordered cone metric spaces. The purpose of this paper is to obtain fixed point and common fixed point of mapppings satisfying a generalized contractive condition than given in [2] in the frame work of ordered cone metric spaces.

Consistent with Huang and Zhang [4], the following definitions and results will be needed in the sequel.

Let E be a real Banach space. A subset P of E is called a *cone* if and only if:

(a) P is closed, non empty and $P \neq \{\theta\}$;
(b) $a, b \in R$, $a, b \geq 0$, $x, y \in P$ imply that $ax + by \in P$;
(c) $P \cap (-P) = \{\theta\}$.

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Given a cone $P \subset E$, we define a partial ordering $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. A cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$$(1.1) \qquad \theta \preceq x \preceq y \text{ implies } \|x\| \leq K \|y\| .$$

The least positive number satisfying the above inequality is called the *normal constant* of P, while $x << y$ stands for $y - x \in intP$ (interior of P).

1.1. Definition. Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies:

(d1) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
(d2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(d3) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X, d) is called a *cone metric space*. The concept of a cone metric space is more general than that of a metric space.

1.2. Definition. Let (X, d) be a cone metric space, $\{x_n\}$ a sequence in X and $x \in X$. For every $c \in E$ with $0 << c$, we say that $\{x_n\}$ is:

(i) a *Cauchy* sequence if there is an N such that, for all $n, m > N$, $d(x_n, x_m) \ll c$;
(ii) a *convergent* sequence if there is an N such that, for all $n > N$, $d(x_n, x) \ll c$ for some x in X.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X. It is known that if the P is normal, then $\{x_n\}$ converges to $x \in X$ if and only if $d(x_n, x) \to \theta$ as $n \to \infty$. The limit of a convergent sequence is unique provided P is a solid cone ($intP \neq \emptyset$) (see, [5, 6, 10]).

1.3. Remark. If E is a real Banach space with a cone P and

(a) if $a \preceq ha$ where $a \in P$ and $h \in [0, 1,$ then $a = \theta$.
(b) If $x \ll y \preceq z$, then $x \ll z$.
(c) If $x \preceq y \ll z$, then $x \ll z$.
(d) If $x \ll y \ll z$, then $x \ll z$.

Let (X, d) be a cone metric space, $f : X \to X$ and $x_0 \in X$. Then the function f is continuous at x_0 if for any sequence $x_n \to x_0$ we have $fx_n \to fx_0$. If $(X, \sqsubseteq)$ is a partially ordered set and $f : X \to X$ is such that $fx \sqsubseteq fy$ whenever $x, y \in X$ and $x \sqsubseteq y$ then f is said to be nondecreasing.

2. Fixed Point Theorems

In this section we obtain results of fixed point theorems for mappings defined on a cone metric space.

2.1. Theorem. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exists a cone metric d on X such that the cone metric space (X, d) is complete. Let $f : X \to X$ be a continuous and nondecreasing mapping with respect to $\sqsubseteq$ which satisfy*

$$(2.1) \qquad d(fx, fy) \preceq hu(x, y)$$

where $h \in (0, 1)$ and

$$u(x, y) \in \{d(x, y), d(x, fx), d(y, fy), \frac{d(x, fx) + d(y, fy)}{2}, \frac{d(x, fy) + d(y, fx)}{2}\}$$

for all x, $y \in X$ with $y \sqsubseteq x$. If there exists $x_0 \in X$ such that $x_0 \sqsubseteq fx_0$, then f has a fixed point in X.

Proof. Since $x_0 \sqsubseteq fx_0$ and f is nondecreasing with respect to $\sqsubseteq$. Therefore

$$x_0 \sqsubseteq fx_0 \sqsubseteq f^2x_0 \sqsubseteq ... \sqsubseteq f^{n-1}x_0 \sqsubseteq f^nx_0 \sqsubseteq f^{n+1}x_0 \sqsubseteq ....$$

Now for any n in N, we have

$$d(f^{n+1}x_0, f^nx_0) \preceq hu(f^nx_0, f^{n-1}x_0) \tag{2.2}$$

where

$$\begin{aligned}u(f^nx_0, f^{n-1}x_0) \in \Big\{&d(f^nx_0, f^{n-1}x_0), d(f^nx_0, f^{n+1}x_0), d(f^{n-1}x_0, f^nx_0),\\ &\frac{d(f^nx_0, f^{n+1}x_0) + d(f^{n-1}x_0, f^nx_0)}{2}, \frac{d(f^nx_0, f^nx_0) + d(f^{n-1}x_0, f^{n+1}x_0)}{2}\Big\}\\ = \Big\{&d(f^nx_0, f^{n-1}x_0), d(f^nx_0, f^{n+1}x_0), \frac{d(f^nx_0, f^{n+1}x_0) + d(f^{n-1}x_0, f^nx_0)}{2},\\ &\frac{1}{2}d(f^{n-1}x_0, f^{n+1}x_0)\Big\}.\end{aligned}$$

Now $u(f^nx_0, f^{n-1}x_0) = d(f^nx_0, f^{n-1}x_0)$, implies that

$$d(f^{n+1}x_0, f^nx_0) \preceq hd(f^nx_0, f^{n-1}x_0).$$

If $u(f^nx_0, f^{n-1}x_0) = d(f^nx_0, f^{n+1}x_0)$, then

$$d(f^{n+1}x_0, f^nx_0) \preceq hd(f^nx_0, f^{n+1}x_0),$$

which by Remark 1.3 (a) implies that $f^{n+1}x_0 = f^nx_0$ and result follows in this case. If $u(f^nx_0, f^{n-1}x_0) = \dfrac{d(f^nx_0, f^{n+1}x_0) + d(f^{n-1}x_0, f^nx_0)}{2}$, then we obtain

$$\begin{aligned}d(f^{n+1}x_0, f^nx_0) &\preceq \frac{h}{2}\{d(f^nx_0, f^{n+1}x_0) + d(f^{n-1}x_0, f^nx_0)\}\\ &\preceq \frac{1}{2}d(f^nx_0, f^{n+1}x_0) + \frac{h}{2}d(f^{n-1}x_0, f^nx_0),\end{aligned}$$

$d(f^{n+1}x_0, f^nx_0) \preceq hd(f^{n-1}x_0, f^nx_0)$. Finally, for $u(f^nx_0, f^{n-1}x_0) = \dfrac{d(f^{n-1}x_0, f^{n+1}x_0)}{2}$, we get

$$\begin{aligned}d(f^{n+1}x_0, f^nx_0) &\preceq \frac{h}{2}d(f^{n-1}x_0, f^{n+1}x_0)\\ &\preceq \frac{h}{2}d(f^{n-1}x_0, f^nx_0) + \frac{h}{2}d(f^nx_0, f^{n+1}x_0)\\ &\preceq \frac{h}{2}d(f^{n-1}x_0, f^nx_0) + \frac{1}{2}d(f^nx_0, f^{n+1}x_0),\end{aligned}$$

which further implies that $d(f^{n+1}x_0, f^nx_0) \preceq hd(f^{n-1}x_0, f^nx_0)$. So

$$d(f^{n+1}x_0, f^nx_0) \preceq hd(f^{n-1}x_0, f^nx_0),$$

for all $n \geq 1$. Repeating above process we get

$$\begin{aligned}d(f^{n+1}x_0, f^nx_0) &\preceq hd(f^{n-1}x_0, f^nx_0) \preceq h^2d(f^{n-2}x_0, f^{n-2}x_0)\\ &\preceq ... \preceq h^nd(fx_0, x_0).\end{aligned}$$

for all $n \in \mathbb{N}$, and so for $m > n$, we have

$$\begin{aligned}d(f^mx_0, f^nx_0) &\preceq d(f^mx_0, f^{m-1}x_0) + ... + d(f^{n+1}x_0, f^nx_0)\\ &\preceq (h^{m-1} + h^{m-2} + ... + h^n)d(fx_0, x_0)\\ &\preceq \frac{h^n}{1-h}d(fx_0, x_0).\end{aligned}$$

Let $0 \ll c$ be given. Choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where $N_\delta(0) = \{y \in E : \|y\| < \delta\}$. Also, choose $N_1 \in \mathbb{N}$ such that $\frac{h^n}{1-h} d(fx_0, x_0) \in N_\delta(0)$, for all $n \geq N_1$ which implies that $\frac{h^n}{1-h} d(fx_0, x_0) \ll c$, for all $n > N_1$ and hence, according to Remark 1.3 (c) we have that

$$d(f^m x_0, f^n x_0) \ll c$$

for all $n, m > N_1$. Therefore $\{f^n x_0\}$ is a Cauchy sequence in X. Since X is complete, there exists an element $x^* \in X$ such that $f^n x_0 \to x^*$ as $n \to \infty$. Now $f(f^n x_0) = f^{n+1} x_0 \to x^*$ implies that $fx^* = x^*$. Hence x^* is a fixed point of f. □

2.2. Corollary. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exists a cone metric d on X such that the cone metric space (X, d) is complete. Let $f : X \to X$ be a continuous and nondecreasing mapping with respect to $\sqsubseteq$ which satisfy*

$$d(fx, fy) \preceq hu(x, y) \tag{2.1}$$

where $h \in (0, 1)$ and

$$u(x, y) \in \{d(x, y), d(x, fx), \frac{d(x, fx) + d(y, fy)}{2}, \frac{d(x, fy) + d(y, fx)}{2}\}$$

for all $x, y \in X$ with $y \sqsubseteq x$. If there exists $x_0 \in X$ such that $x_0 \sqsubseteq fx_0$, then f has a fixed point in X.

2.3. Example. Let $E = C_R[0, \infty)$, $P = \{f \in E : f(t) \geq 0\}$, $X = [0, 1]$ with usual order and with cone metric $d : X \times X \to E$ defined by $d(x, y) = F_{x,y}$, where $F_{x,y}(t) = t\,|x - y|$ for all $t \in [0, \infty)$ ([3]). Define $f : X \to X$ as $f(x) = \frac{1}{3}x$.
Now $d(fx, fy)(t) = F_{fx,fy}(t) = t\,|fx - fy| = \frac{t}{3}\,|x - y|$ and

$$\begin{aligned}
d(x, y)\,(t) &= F_{x,y}(t) = t\,|x - y|\,, \\
d(x, fx)(t) &= F_{x,fx}(t) = \frac{2t}{3}x, \\
\frac{(d(x, fx) + d(y, fy))}{2}(t) &= \frac{F_{x,fx}(t) + F_{y,fy}(t)}{2} = \frac{t}{3}(x + y) \\
\frac{(d(x, fy) + d(y, fx))}{2}(t) &= \frac{F_{x,fy}(t) + F_{y,fx}(t)}{2} = t\frac{(\left|x - \frac{y}{3}\right| + \left|y - \frac{x}{3}\right|)}{2}
\end{aligned}$$

Note that

$$\begin{aligned}
d(fx, fy)(t) &= \frac{t}{3}\,|x - y| \preceq t\,|x - y| = d(x, y)\,(t)\,, \\
d(fx, fy)(t) &= \frac{t}{3}\,|x - y| \preceq \frac{2t}{3}x = d(x, fx)(t), \\
d(fx, fy)(t) &= \frac{t}{3}\,|x - y| \preceq \frac{t}{3}(x + y) = \frac{d(x, fx) + d(y, fy)}{2}(t), \\
d(fx, fy)(t) &= \frac{t}{3}\,|x - y| \preceq t\frac{(\left|x - \frac{y}{3}\right| + \left|y - \frac{x}{3}\right|)}{2} = \frac{(d(x, fy) + d(y, fx))}{2}(t).
\end{aligned}$$

for all $x, y \in X$ with $y \preceq x$. So contractive condition of Corollary 2.2 is satisfied. Moreover 0 is the fixed point of f.

2.4. Definition ([2])**.** Let $(X, \sqsubseteq)$ be a partially ordered set. Two mappings $f, g : X \to X$ are said to be weakly increasing if $fx \sqsubseteq gfx$ and $gx \sqsubseteq fgx$ for all $x \in X$.

The following two examples shows that there exist discontinous not nondecreasing mappings which are weakly increasing.

2.5. Example. Let $X = (0,\infty)$, endowed with usual ordering. Let $f, g : X \to X$ be defined by

$$fx = \begin{cases} 3x+2 \text{ if } 0 < x < 1 \\ 2x+1 \text{ if } 1 \le x < \infty \end{cases}$$

and

$$gx = \begin{cases} 4x+1 \text{ if } 0 < x < 1 \\ 3x \text{ if } 1 \le x < \infty \end{cases}.$$

For $0 < x < 1$, $fx = 3x+2 \le 3(3x+2) = gfx$ and $gx = 4x+1 \le 4x+3 = 2(2x+1)+1 = fgx$ and for $1 \le x < \infty$, $fx = 2x+1 \le 3(2x+1) = gfx$ and $gx = 3x \le 2(3x)+1 = fgx$. Thus f and g are weakly increasing maps but not nondecreasing.

2.6. Example. Let $X = [0,\infty)\times[0,\infty)$ with the usual ordering, that is, $(x,y) \lesssim (z,w)$, iff $x \le z$ and $y \le w$. Let $f, g : X \to X$ be defined by

$$f(x,y) = \begin{cases} (x,y) \text{ if } \max\{x,y\} \le 1 \\ (0,0) \text{ if } \max\{x,y\} > 1 \end{cases}$$

and

$$g(x,y) = \begin{cases} (\sqrt{x},\sqrt{y}) \text{ if } \max\{x,y\} \le 1 \\ (0,0) \text{ if } \max\{x,y\} > 1 \end{cases}.$$

For $\max\{x,y\} \le 1$, $f(x,y) = (x,y) \lesssim (\sqrt{x},\sqrt{y}) = gf(x,y)$ and $g(x,y) = (\sqrt{x},\sqrt{y}) \lesssim (\sqrt{x},\sqrt{y}) = fg(x,y)$ and for $\max\{x,y\} > 1$, $f(x,y) = g(x,y) = (0,0) \lesssim fg(x,y) = gf(x,y)$. Thus f and g are weakly increasing mappings. Also note that both f and g are not nondecreasing. For example, $(\frac{1}{2},1) \lesssim (1,2)$ but $f(\frac{1}{2},1) = (\frac{1}{2},1) \not\lesssim (0,0) = f(1,2)$.

2.7. Theorem. *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exists a cone metric d on X such that the cone metric space (X,d) is complete. Let $f, g : X \to X$ be two weakly increasing mappings with respect to $\sqsubseteq$ which satisfy*

$$d(fx,gy) \preceq hu(x,y) \tag{2.3}$$

where $h \in (0,1)$ and

$$u(x,y) \in \{d(x,y), d(x,fx), d(y,gy), \frac{d(x,fx)+d(y,gy)}{2}, \frac{d(x,gy)+d(y,fx)}{2}\}$$

for all comparative x, $y \in X$. Then f and g have a common fixed point in X provided f or g is continuous.

Proof. Suppose x_0 is an arbitrary point of X and $\{x_n\}$ a sequence in X such that $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for all $n \ge 0$. Since f and g are weakly increasing therefore $x_1 = fx_0 \sqsubseteq gfx_0 = gx_1 = x_2 = gx_1 \sqsubseteq fgx_1 = fx_2 = x_3$ and continuing this process we have $x_1 \sqsubseteq x_2 \sqsubseteq ... \sqsubseteq x_n \sqsubseteq x_{n+1} \sqsubseteq ....$ That is, the sequence $\{x_n\}$ is nondecreasing. Since x_{2n} and x_{2n+1} are comparative, therefore

$$d(x_{2n+1},x_{2n+2}) = d(fx_{2n},gx_{2n+1}) \preceq hu(x_{2n},x_{2n+1}) \tag{2.4}$$

where

$$\begin{aligned} u(x_{2n},x_{2n+1}) \in \{&d(x_{2n},x_{2n+1}), d(x_{2n},x_{2n+1}), d(x_{2n+1},x_{2n+2}), \\ &\frac{d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})}{2}, \frac{d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1})}{2}\} \\ &= \{d(x_{2n},x_{2n+1}), d(x_{2n+1},x_{2n+2}), \\ &\frac{d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})}{2}, \frac{d(x_{2n},x_{2n+2})}{2}\}. \end{aligned}$$

Now $u(x_{2n}, x_{2n+1}) = d(x_{2n}, x_{2n+1})$ implies that

$$d(x_{2n+1}, x_{2n+2}) \preceq hd(x_{2n}, x_{2n+1}).$$

If $u(x_{2n}, x_{2n+1}) = d(x_{2n+1}, x_{2n+2})$, then

$$d(x_{2n+1}, x_{2n+2}) \preceq hd(x_{2n+1}, x_{2n+2}),$$

which by Remark 1.3 (a) implies that $x_{2n+1} = x_{2n+2}$ and the result follows in this case. If $u(x_{2n}, x_{2n+1}) = \dfrac{d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})}{2}$ then we obtain

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &\preceq \frac{h}{2}\left(d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})\right) \\ &\preceq \frac{h}{2}d(x_{2n}, x_{2n+1}) + \frac{1}{2}d(x_{2n+1}, x_{2n+2}), \end{aligned}$$

which further implies that

$$d(x_{2n+1}, x_{2n+2}) \preceq hd(x_{2n}, x_{2n+1}).$$

Finally, $u(x_{2n}, x_{2n+1}) = \dfrac{d(x_{2n}, x_{2n+2})}{2}$ gives that

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &\preceq \frac{h}{2}d(x_{2n}, x_{2n+2}) \preceq \frac{h}{2}\left(d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})\right) \\ &\preceq \frac{h}{2}d(x_{2n}, x_{2n+1}) + \frac{1}{2}d(x_{2n+1}, x_{2n+2}), \end{aligned}$$

which implies that $d(x_{2n+1}, x_{2n+2}) \preceq hd(x_{2n}, x_{2n+1})$. So we conclude that

$$d(x_{2n+1}, x_{2n+2}) \preceq hd(x_{2n}, x_{2n+1})$$

for all $n \geq 1$ and consequently

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &\preceq hd(x_{2n}, x_{2n+1}) \preceq h^2 d(x_{2n-1}, x_{2n}) \\ &\preceq \ldots \preceq h^{2n} d(x_0, x_1). \end{aligned}$$

for all $n \in \mathbb{N}$. Now for $m > n$, we have

$$\begin{aligned} d(x_m, x_n) &\preceq d(x_m, x_{m-1}) + \ldots + d(x_{n+1}, x_n) \\ &\preceq (h^{m-1} + h^{m-2} + \ldots + h^n)d(x_1, x_0) \\ &\preceq \frac{h^n}{1-h}d(x_1, x_0). \end{aligned}$$

Let $0 \ll c$ be given. Choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where $N_\delta(0) = \{y \in E : \|y\| < \delta\}$. Also, choose $N_1 \in \mathbb{N}$ such that $\dfrac{h^n}{1-h}d(x_1, x_0) \in N_\delta(0)$, for all $n \geq N_1$ which implies that $\dfrac{h^n}{1-h}d(x_1, x_0) \ll c$, for all $n > N_1$ and hence, according to Remark 1.3 (c) we have that

$$d(x_m, x_n) \ll c$$

for all $n, m > N_1$. Therefore $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there exists an element $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$.
Suppose that f is continuous then $f(f^n x_0) = f^{n+1}x_0 \to x^*$ implies that $fx^* = x^*$. Hence x^* is a fixed point of f. Since $x^* \sqsubseteq x^*$ therefore

$$d(fx^*, gx^*) \preceq hu(x^*, x^*)$$

where

$$\begin{aligned} u(x^*, x^*) &\in \{d(x^*, x^*), d(x^*, fx^*), d(x^*, gx^*), \\ &\quad \frac{d(x^*, fx^*) + d(x^*, gx^*)}{2}, \frac{d(x^*, gx^*) + d(x^*, fx^*)}{2}\} \\ &= \{d(x^*, gx^*), \frac{1}{2}d(x^*, gx^*)\}. \end{aligned}$$

Now $u(x^*, x^*) = d(x^*, gx^*)$ implies that

$$d(x^*, gx^*) \preceq hd(x^*, gx^*),$$

which by Remark 1.3 (a) implies that $gx^* = x^*$.
If $u(x^*, x^*) = \frac{1}{2}d(x^*, gx^*)$, then

$$d(x^*, gx^*) \preceq \frac{h}{2}d(x^*, gx^*),$$

so again by Remark 1.3 (a) implies that $gx^* = x^*$. So f and g have a common fixed point in X. □

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