

INTRODUCTION TO GENERALIZED SPATIAL LOCALES

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Abstract

We review the notion of generalized topological space and introduce generalized spatial locales (*gs-locales*) and their density, describe homomorphisms and isomorphisms of *gs-locales*, provide representation theorems for generalized topological spaces and *gs-locales* and show the categorical relations between *gs-locales* and T_0 topological spaces.

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1. Motivation

The concept of *topological space modulo ideal* was introduced in [4]. Later we developed the notion of *generalized topological space (gt-space for short)* [5]. The generalized topology of a *gt-space* is a frame. The join and meet operations of a generalized topology may not coincide with the usual union and intersection operations for sets, in fact the join and meet operations are union and intersection operations modulo *small sets*. The family of small sets of a *gt-space* possesses the structure of an *ideal* and contains no open sets, the only exception being the empty set which is both open and small.

In order to better understand the nature of generalized topological spaces we decided to look at them from the point of view of locale theory.

The second motivation for our research follows from the following observation. Locale theory study the isomorphism between the category of *sober spaces* and the category of *spatial locales* [3]. But there exist topological spaces that are not sober and frames that are not spatial. In Section 2, we provide the right arrow generalized topological space. Obviously, this space has a nontrivial topological structure. On the other hand, the generalized topology of this space is a frame but not a spatial frame and, moreover, the family of all its principal prime ideals [3] is empty, which means this frame is not

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