

SURFACES IN THE EUCLIDEAN SPACE \mathbb{E}^4 WITH POINTWISE 1-TYPE GAUSS MAP

Uğur Dursun* and Güler Gürpınar Arsan*†

Received 15:09:2009 : Accepted 02:03:2011

Abstract

In this article we study surfaces in Euclidean space \mathbb{E}^4 with pointwise 1-type Gauss map. We give a characterization of surfaces in \mathbb{E}^4 with a pointwise 1-type Gauss map of the first kind. We conclude that an oriented non-minimal surface M in \mathbb{E}^4 has a pointwise 1-type Gauss map of the first kind if and only if M is a surface in a 3-sphere of \mathbb{E}^4 with constant mean curvature. We also obtain a characterization for non-planar minimal surfaces in \mathbb{E}^4 with pointwise 1-type Gauss map of the second kind. Further we give a partial classification of surfaces in \mathbb{E}^4 in terms of the pointwise 1-type Gauss map of the second kind.

Keywords: Minimal surface, Normal bundle, Mean curvature, Pointwise 1-type, Gauss map.

2000 AMS Classification: 53B25, 53C40.

1. Introduction

A submanifold M of a Euclidean space E^m is said to be of *finite type* if its position vector x can be expressed as a finite sum of eigenvectors of the Laplacian Δ of M , that is, $x = x_0 + x_1 + \cdots + x_k$, where x_0 is a constant map, x_1, \dots, x_k are non-constant maps such that $\Delta x_i = \lambda_i x_i$, $\lambda_i \in \mathbb{R}$, $i = 1, 2, \dots, k$.

If $\lambda_1, \lambda_2, \dots, \lambda_k$ are all different, then M is said to be of k -type (cf. [7, 8]). In [9], this definition was similarly extended to differentiable maps, in particular, to Gauss maps of submanifolds.

The notion of a finite type Gauss map is especially a useful tool in the study of submanifolds (cf. [2, 3, 4, 5, 9, 16]). In [9], Chen and Piccinni made a general study on

*Istanbul Technical University, Faculty of Science and Letters, Department of Mathematics, 34469 Maslak, Istanbul, Turkey.

E-mail: (U. Dursun) udursun@itu.edu.tr (G. G. Arsan) ggarsan@itu.edu.tr

†Corresponding Author.