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A RECURRENCE RELATION FOR BERNOULLI NUMBERS

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Abstract

Inspired by a result of Saalschütz, we prove a recurrence relation for Bernoulli numbers. This recurrence relation has an interesting connection with real cyclotomic fields.

Keywords: Bernoulli numbers, cyclotomic fields, Newton's identities.

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1. Introduction

The Bernoulli numbers B_n, which can be defined by the Laurent series expansion

$$\frac{x}{e^x - 1} = \sum_{j=0}^{\infty} \frac{B_j}{j!} x^j,$$

have several applications in different fields of mathematics, such as number theory, combinatorics, numerical analysis.

A classical problem in elementary number theory is to find formulas for summing the m-th powers of the first $n-1$ integers. The following result is obtained by Bernoulli [7, Chap. 15] and it is one of the most historical formula including Bernoulli numbers

$$\sum_{j=1}^{n-1} j^m = \frac{1}{m+1} \sum_{j=0}^{m} \binom{m+1}{j} B_j n^{m+1-j}. \tag{1.1}$$

Another historical formula including Bernoulli numbers is the Euler-Maclaurin summation formula which was found by Euler and Maclaurin independently in 1730s and used for computations in numerical analysis [2], [9]. A breakthrough in algebraic number theory is the Kummer's criterion, proved in 1850, which relates the numerator of Bernoulli numbers to the existence of integer solutions of Fermat's equation [8].

It is a powerful method to investigate Bernoulli numbers by recurrence relations. For example, in order to prove von Staudt-Clausen theorem, concerning the denominator of the Bernoulli numbers, it is important to write B_j in terms of previous Bernoulli numbers

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