SOME MATRIX TRANSFORMATIONS
ON SEQUENCE SPACES OF
INVARIANT MEANS

Mursaleen*

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Abstract
In this paper we define new sequence spaces $V_\sigma(\theta)$ and $V_\infty^\sigma(\theta)$ which are related to the concept of $\sigma$-mean and lacunary sequence $\theta = (k_r)$, and characterize the matrix classes $(l_1, V_\infty^\sigma(\theta))$ and $(l_\infty, V_\infty^\sigma(\theta))$.

Keywords: Lacunary sequence, Matrix transformation, Invariant mean, Almost lacunary convergence.

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1. Introduction and preliminaries
We shall write $w$ for the set of all complex sequences $x = (x_k)^\infty_{k=0}$. Let $\varphi$, $l_\infty$, $c$ and $c_0$ denote the sets of all finite, bounded, convergent and null sequences respectively. We write $l_p := \{x \in w : \sum_{k=0}^{\infty} |x_k|^p < \infty\}$ for $1 \leq p < \infty$. By $e$ and $e^{(n)} (n \in \mathbb{N})$, we denote the sequences such that $e_k = 1$ for $k = 0, 1, \ldots$, $e_n^{(n)} = 1$ and $e_k^{(n)} = 0 (k \neq n)$. For any sequence $x = (x_k)^\infty_{k=0}$, let $x^{[n]} = \sum_{k=0}^{\infty} x_k e^{(k)}$ be its n-section.

Note that $c_0$, $c$, and $l_\infty$ are Banach spaces with the sup-norm $\|x\|_\infty = \sup_{k} |x_k|$, and $l_p^p (1 \leq p < \infty)$ are Banach spaces with the norm $\|x\|_p = (\sum |x_k|^p)^{1/p}$ while $\varphi$ is not a Banach space with respect to any norm.

A sequence $(b^{(n)})^\infty_{n=0}$ in a linear metric space $X$ is called a Schauder basis if for every $x \in X$ there is a unique sequence $(\beta_n)^\infty_{n=0}$ of scalars such that $x = \sum_{n=0}^{\infty} \beta_n b^{(n)}$. A sequence space $X$ with a linear topology is called a K-space if each of the maps $p_i : X \rightarrow \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for all $i \in \mathbb{N}$. A K-space is called an FK-space if $X$ is a complete linear metric space, and a BK-space is a normed FK-space. An FK-space $X \supset \varphi$ is said to have an AK if every sequence $x = (x_k)^\infty_{k=0} \in X$ has a unique representation $x = \sum_{k=0}^{\infty} x_k e^{(k)}$, that is, $x = \lim_{n \rightarrow \infty} x^{[n]}$. We use here standard notations as in [7].

*Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.
E-mail: mursaleenm@gmail.com