\textbf{\textit{X}-GORENSTEIN PROJECTIVE AND \\
\textit{Y}-GORENSTEIN INJECTIVE MODULES}

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Abstract

Let $\mathcal{X}$ be a class of right $R$-modules that contains all projective right $R$-modules. The notion of $\mathcal{X}$-Gorenstein projective modules was introduced by D. Bennis and K. Ouarghi ($\mathcal{X}$-Gorenstein projective modules, International Mathematical Forum \textbf{5}(10), 487–491, 2010). In this paper, we introduce $\mathcal{Y}$-Gorenstein injective right $R$-modules and $\mathcal{Y}$-Gorenstein flat left $R$-modules, where $\mathcal{Y}$ is a class of right $R$-modules that contains all injective right $R$-modules. We show that the principal results on Gorenstein modules remain true for $\mathcal{X}$-Gorenstein projective right $R$-modules, $\mathcal{Y}$-Gorenstein injective right $R$-modules and $\mathcal{Y}$-Gorenstein flat left $R$-modules.

\textbf{Keywords:} \ $\mathcal{X}$-Gorenstein projective modules, $\mathcal{Y}$-Gorenstein injective modules, $\mathcal{Y}$-Gorenstein flat modules.

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1. Introduction

In [6], Enochs and Jenda defined the Gorenstein injective modules over an arbitrary ring $R$. Recall that a right $R$-module $M$ is called Gorenstein injective if there is an exact sequence

$$E \equiv \ldots \to E^{-2} \to E^{-1} \to E^0 \to E^1 \to \ldots$$

of injective right $R$-modules with $M = \ker(E^0 \to E^1)$, and which remains exact whenever $\text{Hom}_R(E, -)$ is applied for any injective right $R$-module $E$. 

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