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RESULTS ON THE COMPOSITION AND NEUTRIX COMPOSITION OF THE DELTA FUNCTION

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Abstract

The neutrix composition $F(f(x)))$ of a distribution $F(x)$ and a locally summable function $f(x)$ is said to exist and be equal to the distribution $h(x)$ if the neutrix limit of the sequence $\{F_n(f(x))\}$ is equal to $h(x)$, where $F_n(x) = F(x) * \delta_n(x)$ and $\{\delta_n(x)\}$ is a certain sequence of infinitely differentiable functions converging to the Dirac delta-function $\delta(x)$. It is proved that the neutrix composition $\delta^{(s)}\{[\exp_+(x) - 1]^r\}$ exists and

$$\delta^{(s)}\{[\exp_+(x) - 1]^r\} = \sum_{k=0}^{rs+r-1} \frac{(-1)^{s+k} s! c_{rs+r-1,k}}{2rk!} \delta^{(k)}(x),$$

for $r = 1, 2, \ldots$ and $s = 0, 1, 2, \ldots$. Further results are also proved.

Keywords: distribution, dirac-delta function, composition of distributions, neutrix, neutrix limit.

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1. Introduction

Certain operations on smooth functions (such as addition, and multiplication by scalars) can be extended without difficulty to arbitrary distributions. Others (such as multiplication, convolution, and change of variables) can be defined only for particular distributions. Note that it is a difficult task to give a meaning to the expression $F(f(x))$, if F and f are singular distributions.

The technique of neglecting appropriately defined infinite quantities was devised by Hadamard and the resulting finite value extracted from the divergent integral is usually referred to as the Hadamard finite part. In fact, Hadamard's method can be regarded

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