

ON A FUNCTIONAL EQUATION ORIGINATING FROM A MIXED ADDITIVE AND CUBIC EQUATION AND ITS STABILITY

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Abstract

In this paper, we study solutions of the 2-variable mixed additive and cubic functional equation

$$f(2x + y, 2z + t) + f(2x - y, 2z - t) = 2f(x + y, z + t) \\ + 2f(x - y, z - t) + 2f(2x, 2z) - 4f(x, z),$$

which has the cubic form $f(x, y) = ax^3 + bx^2y + cxy^2 + dy^3$ as a solution. Also the Hyers–Ulam–Rassias stability of this equation in the non-Archimedean Banach spaces is investigated.

Keywords: Hyers–Ulam–Rassias stability, Cubic functional equation, Non–Archimedean normed space, Derivation.

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1. Introduction and preliminaries

The study of stability problems for functional equations is related to a question of Ulam [26] concerning the stability of group homomorphisms, affirmatively answered for Banach spaces by Hyers [8]. Subsequently, the result of Hyers was generalized by Aoki [1], Bourgin [5] and Rassias [24].

During the last decades several stability problems for various functional equations have been investigated by numerous mathematicians. We refer the reader to the survey articles [7, 9, 23] and monographs [6, 10, 13, 22] and references therein.

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