ON SOME INEQUALITIES OF SIMPSON’S TYPE VIA $h$–CONVEX FUNCTIONS

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Abstract

In this paper, we prove some new inequalities of Simpson’s type for functions whose derivatives of absolute values are $h$–convex and $h$–concave functions. Some new estimations are obtained. Also we give some sophisticated results for some different kinds of convex functions.

Keywords: $h$–convex and $h$–concave functions, Simpson’s Inequality, Hölder Inequality.

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1. Introduction

The following inequality is well known in the literature as Simpson’s inequality:

$$\frac{1}{b-a} \int_{a}^{b} f(x) \, dx - \frac{1}{3} \left[ \frac{f(a) + f(b)}{2} + 2f\left(\frac{a + b}{2}\right) \right] \leq \frac{1}{2880} \|f^{(4)}\|_{\infty} (b - a)^{4},$$

where the mapping $f : [a, b] \to \mathbb{R}$ is assumed to be four times continuously differentiable on the interval and $f^{(4)}$ to be bounded on $(a, b)$, that is,

$$\|f^{(4)}\|_{\infty} = \sup_{t \in (a, b)} |f^{(4)}(t)| < \infty.$$

For some results which generalize, improve and extend the inequality (1.1) see the papers [1]–[3].

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