A GENERALIZATION OF REDUCED RINGS

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Abstract

Let \( R \) be a ring with identity. We introduce a class of rings which is a generalization of reduced rings. A ring \( R \) is called central rigid if for any \( a, b \in R \), \( a^2b = 0 \) implies \( ab \) belongs to the center of \( R \). Since every reduced ring is central rigid, we study sufficient conditions for central rigid rings to be reduced. We prove that some results of reduced rings can be extended to central rigid rings for this general setting, in particular, it is shown that every reduced ring is central rigid, every central rigid ring is central reversible, central semicommutative, 2-primal, abelian and so directly finite.

Keywords: Reduced rings, Central rigid rings, Central reversible rings, Central semicommutative rings, Abelian rings.

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1. Introduction

Throughout this paper all rings are associative with identity unless otherwise stated. A ring is reduced if it has no nonzero nilpotent elements. Recently the reduced ring concept was extended to \( R \)-modules by Lee and Zhou in [13], that is, an \( R \)-module \( M \) is called reduced if, for any \( m \in M \) and any \( a \in R \), \( ma = 0 \) implies \( mR \cap Ma = 0 \). According to Cohn [9] a ring \( R \) is called reversible if for any \( a, b \in R \), \( ab = 0 \) implies \( ba = 0 \). A ring \( R \) is called central reversible if for any \( a, b \in R \), \( ab = 0 \) implies \( ba \) belongs to the center of \( R \). A ring \( R \) is called semicommutative if for any \( a, b \in R \), \( ab = 0 \) implies \( aRb = 0 \), while the ring \( R \) is said to be central semicommutative [3] if for any \( a, b \in R \), \( ab = 0 \) implies \( arb \) is a central element of \( R \) for each \( r \in R \) and \( arb \) is a central element of \( R \) for each \( r \in R \). A ring \( R \) is called right (left) principally quasi-Baer [8] if the right (left) annihilator of a principal right ideal of \( R \) is generated by an idempotent. Finally, a ring \( R \) is called right (left) principally projective if the right (left) annihilator of an element of \( R \) is generated by an idempotent [7]. For a positive integer \( n \), \( \mathbb{Z}_n \) denotes the ring of integers modulo \( n \). We write \( R[x] \) and \( R[x, x^{-1}] \) for the polynomial ring and the Laurent polynomial ring, respectively.

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