ON GENERALIZED DERIVATIONS OF PRIME NEAR-RINGS

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Abstract

Let $N$ be a 2–torsion free prime near-ring with center $Z$, $(f, d)$ and $(g, h)$ two generalized derivations on $N$. In this case: (i) If $f([x, y]) = 0$ or $f([x, y]) = [x, y]$ or $f^2(x) \in Z$ for all $x, y \in N$, then $N$ is a commutative ring. (ii) If $a \in N$ and $[f(x), a] = 0$ for all $x \in N$, then $d(a) \in Z$. (iii) If $(f g, dh)$ acts as a generalized derivation on $N$, then $f = 0$ or $g = 0$.

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1. Introduction

Throughout this paper $N$ will denote a zero symmetric left near-ring with multiplicative centre $Z$. Recall that a near-ring $N$ is prime if $xNy = 0$ implies $x = 0$ or $y = 0$. An additive mapping $d : N \rightarrow N$ is said to be a derivation on $N$ if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$ or equivalently, as noted in [3], that $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$. Further an element $x \in N$ for which $d(x) = 0$ is called a constant. For $x, y \in N$ the symbol $[x, y]$ will denote the commutator $xy - yx$, while the symbol $(x, y)$ will denote the additive-group commutator $x + y - y - x$.

Over the last two decades, a lot of work has been done on commutativity of prime rings with derivation. It is natural to look for comparable results on near-rings and this has been done [1,3,4] (where further references can be found). Recently, in [5], Bresar defined the following notation:

An additive mapping $f : R \rightarrow R$ is called a generalized derivation if there exits a derivation $d$ of $R$ such that

$$f(xy) = f(x)y + xd(y)$$

for all $x, y \in R$.

The concept of generalized derivation cover also the concept of a derivation. In the present paper we extend some well-known results concerning derivations of prime rings to generalized derivations of prime near-rings.