

ON THE DERIVATION OF EXPLICIT FORMULAE FOR SOLUTIONS OF THE WAVE EQUATION IN HYPERBOLIC SPACE

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Abstract

We offer a new approach to solving the initial value problem for the wave equation in hyperbolic space in arbitrary dimensions. Our approach is based on the spectral analysis of the Laplace-Beltrami operator in hyperbolic space and some structural formulae for rapidly decreasing functions of this operator.

Keywords: Hyperbolic space, Laplace-Beltrami operator, wave equation, spectral projection.

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1. Introduction

The n -dimensional hyperbolic space H^n can be realized as the set

$$(1.1) \quad H^n = \{z = (x_1, \dots, x_{n-1}, y) : -\infty < x_j < \infty (1 \leq j \leq n-1), 0 < y < \infty\}.$$

The H^n is a homogeneous space of the group

$$(1.2) \quad G = SO^+(1, n) = \left\{g \in GL(n+1, \mathbb{R}) : g^T J g = J, \det g = 1, g_{00} > 0\right\},$$

where $GL(n+1, \mathbb{R})$ is the group of all nonsingular real $(n+1) \times (n+1)$ matrices $g = [g_{jk}]_{j,k=0}^n$, J is the $(n+1) \times (n+1)$ diagonal matrix whose the first diagonal element equals -1 and the remaining diagonal elements are all equal to 1 ; the symbol T stands for the matrix transposition.

The group $G = SO^+(1, n)$ acts in H^n as follows: If $g \in G$, $g = [g_{jk}]_{j,k=0}^n$ and $z = (x_1, \dots, x_{n-1}, y)$, then the point

$$gz = z' = (x'_1, \dots, x'_{n-1}, y')$$

has the coordinates

$$(1.3) \quad x'_j = \frac{(g_{j0} + g_{jn})|z|^2 + 2 \sum_{k=1}^{n-1} g_{jk} x_k + g_{j0} - g_{jn}}{c_g |z|^2 + 2 \sum_{k=1}^{n-1} (g_{0k} - g_{nk}) x_k + d_g} \quad (1 \leq j \leq n-1),$$

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