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ON π-MORPHIC MODULES

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Abstract

Let R be an arbitrary ring with identity and M be a right R-module with $S = \mathrm{End}(M_R)$. Let $f \in S$. f is called π-morphic if $M/f^n(M) \cong r_M(f^n)$ for some positive integer n. A module M is called π-morphic if every $f \in S$ is π-morphic. It is proved that M is π-morphic and image-projective if and only if S is right π-morphic and M generates its kernel. S is unit-π-regular if and only if M is π-morphic and π-Rickart if and only if M is π-morphic and dual π-Rickart. M is π-morphic and image-injective if and only if S is left π-morphic and M cogenerates its cokernel.

Keywords: Endomorphism rings; π-morphic rings; π-morphic modules; unit π-regular rings.

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1. Introduction

Throughout this paper all rings have an identity, all modules considered are unital right modules and all ring homomorphisms are unital (unless explicitly stated otherwise).

A ring R is said to be *strongly π-regular* (*π-regular, right weakly π-regular*) if for every element $x \in R$ there exists an integer $n > 0$ such that $x^n \in x^{n+1}R$ (respectively $x^n \in x^nRx^n$, $x^n \in x^nRx^nR$). It is called *unit-π-regular* if for every $a \in R$, there exist a unit element $x \in R$ and a positive integer n such that $a^n = a^nxa^n$. In the case of n = 1 there exists a unit x such that $a = axa$ for all $a \in R$, then R is *unit regular*. Clearly, a strongly π-regular ring is a π-regular ring.

We say also that the ring R is (von Neumann) *regular* if for each $a \in R$ there exists $x \in R$ such that $a = axa$ for some element x in R, that is, a is regular.

A module M is said to satisfy Fitting's lemma if, for all $f \in S$, there exists an integer $n \geq 1$, depending on f, such that $M = f^nM \oplus \mathrm{Ker}(f^n)$. Hence a module satisfies

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