

## ON $\pi$ -MORPHIC MODULES

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### Abstract

Let  $R$  be an arbitrary ring with identity and  $M$  be a right  $R$ -module with  $S = \text{End}(M_R)$ . Let  $f \in S$ .  $f$  is called  $\pi$ -morphic if  $M/f^n(M) \cong r_M(f^n)$  for some positive integer  $n$ . A module  $M$  is called  $\pi$ -morphic if every  $f \in S$  is  $\pi$ -morphic. It is proved that  $M$  is  $\pi$ -morphic and image-projective if and only if  $S$  is right  $\pi$ -morphic and  $M$  generates its kernel.  $S$  is unit- $\pi$ -regular if and only if  $M$  is  $\pi$ -morphic and  $\pi$ -Rickart if and only if  $M$  is  $\pi$ -morphic and dual  $\pi$ -Rickart.  $M$  is  $\pi$ -morphic and image-injective if and only if  $S$  is left  $\pi$ -morphic and  $M$  cogenerates its cokernel.

**Keywords:** Endomorphism rings;  $\pi$ -morphic rings;  $\pi$ -morphic modules; unit  $\pi$ -regular rings.

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### 1. Introduction

Throughout this paper all rings have an identity, all modules considered are unital right modules and all ring homomorphisms are unital (unless explicitly stated otherwise).

A ring  $R$  is said to be *strongly  $\pi$ -regular* ( *$\pi$ -regular*, *right weakly  $\pi$ -regular*) if for every element  $x \in R$  there exists an integer  $n > 0$  such that  $x^n \in x^{n+1}R$  (respectively  $x^n \in x^n R x^n$ ,  $x^n \in x^n R x^n R$ ). It is called *unit- $\pi$ -regular* if for every  $a \in R$ , there exist a unit element  $x \in R$  and a positive integer  $n$  such that  $a^n = a^n x a^n$ . In the case of  $n = 1$  there exists a unit  $x$  such that  $a = axa$  for all  $a \in R$ , then  $R$  is *unit regular*. Clearly, a strongly  $\pi$ -regular ring is a  $\pi$ -regular ring.

We say also that the ring  $R$  is (von Neumann) *regular* if for each  $a \in R$  there exists  $x \in R$  such that  $a = axa$  for some element  $x$  in  $R$ , that is,  $a$  is regular.

A module  $M$  is said to satisfy Fitting's lemma if, for all  $f \in S$ , there exists an integer  $n \geq 1$ , depending on  $f$ , such that  $M = f^n M \oplus \text{Ker}(f^n)$ . Hence a module satisfies

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