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GENERALIZED NOTION OF WEAK MODULE AMENABILITY

Abasalt Bodaghi*

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Abstract

In the present paper, we introduce a new notion of weak module amenability for Banach algebras which is related to module homomorphisms. Among other results, we investigate the relationship between this concept for a Banach algebra $\mathcal{A}$ which is a Banach $\mathfrak{A}$-bimodule with compatible actions, and the quotient Banach algebra $\mathcal{A}/J$ where J is the closed ideal of $\mathcal{A}$ generated by elements of the form $(a\cdot\alpha)b - a(\alpha\cdot b)$ for $a \in \mathcal{A}$ and $\alpha \in \mathfrak{A}$. We then study this concept for an inverse semigroup S, where some examples on $\ell^1(S)$ and $C^*(S)$ are given.

Keywords: Banach modules; Module derivation; Weak amenability; Weak module amenability; Inverse semigroup.

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1. Introduction

Let S be a (discrete) semigroup. The semigroup algebra $\ell^1(S)$ is the Banach algebra consisting of all absolutely summable complex-valued functions on S, with the convolution product and the ℓ^1-norm; $\|f\|_1 = \sum_{s\in S} |f(s)|$ $(f \in \ell^1(S))$. We will use δ_s to denote the point mass function at s; $\delta_s(t) = 1$ if $t = s$ and $= 0$ elsewhere. Using point masses we may represent a function f on S as $f = \sum_{s\in S} f(s)\delta_s$. Here we recall that an *inverse semigroup* is a discrete semigroup S such that for each $s \in S$, there is a unique element $s^* \in S$ with $ss^*s = s$ and $s^*ss^* = s^*$. The set of elements of the form s^*s are called *idempotents* of S and denoted by E.

The concept of amenability for a Banach algebra $\mathcal{A}$ was introduced by B. E. Johnson in [18]. A Banach algebra $\mathcal{A}$ is *amenable* if every bounded derivation from $\mathcal{A}$ into any dual Banach $\mathcal{A}$-module is inner, equivalently if $H^1(\mathcal{A}, X^*) = \{0\}$ for every Banach $\mathcal{A}$-module X, where $H^1(\mathcal{A}, X^*)$ is the *first Hochschild cohomology group* of $\mathcal{A}$ with coefficients in X^*, the first dual space of X. Also, a Banach algebra $\mathcal{A}$ is *weakly amenable* if $H^1(\mathcal{A}, \mathcal{A}^*) = \{0\}$. Bade, Curtis and Dales introduced the notion of weak amenability in [5]. They considered this concept only for commutative Banach algebras. After that

* Department of Mathematics, Garmsar Branch, Islamic Azad University, Garmsar, Iran, Email: abasalt.bodaghi@gmail.com