GENERALIZED NOTION OF WEAK MODULE AMENABILITY

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Abstract

In the present paper, we introduce a new notion of weak module amenability for Banach algebras which is related to module homomorphisms. Among other results, we investigate the relationship between this concept for a Banach algebra $A$ which is a Banach $A$-bimodule with compatible actions, and the quotient Banach algebra $A/J$ where $J$ is the closed ideal of $A$ generated by elements of the form $(a \cdot \alpha)b - a(\alpha \cdot b)$ for $a \in A$ and $\alpha \in A$. We then study this concept for an inverse semigroup $S$, where some examples on $\ell^1(S)$ and $C^*(S)$ are given.

Keywords: Banach modules; Module derivation; Weak amenability; Weak module amenability; Inverse semigroup.

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1. Introduction

Let $S$ be a (discrete) semigroup. The semigroup algebra $\ell^1(S)$ is the Banach algebra consisting of all absolutely summable complex-valued functions on $S$, with the convolution product and the $\ell^1$-norm; $\|f\|_1 = \sum_{s \in S} |f(s)|$ ($f \in \ell^1(S)$). We will use $\delta_s$ to denote the point mass function at $s$; $\delta_s(t) = 1$ if $t = s$ and $= 0$ elsewhere. Using point masses we may represent a function $f$ on $S$ as $f = \sum_{s \in S} f(s) \delta_s$. Here we recall that an inverse semigroup is a discrete semigroup $S$ such that for each $s \in S$, there is a unique element $s^* \in S$ with $ss^*s = s$ and $s^*ss^* = s^*$. The set of elements of the form $s^*s$ are called idempotents of $S$ and denoted by $E$.

The concept of amenability for a Banach algebra $A$ was introduced by B. E. Johnson in [18]. A Banach algebra $A$ is amenable if every bounded derivation from $A$ into any dual Banach $A$-module is inner, equivalently if $H^1(A, X^*) = \{0\}$ for every Banach $A$-module $X$, where $H^1(A, X^*)$ is the first Hochschild cohomology group of $A$ with coefficients in $X^*$, the first dual space of $X$. Also, a Banach algebra $A$ is weakly amenable if $H^1(A, A^*) = \{0\}$. Bade, Curtis and Dales introduced the notion of weak amenability in [5]. They considered this concept only for commutative Banach algebras. After that

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