SOME RESULTS RELATED TO A CERTAIN VECTOR FIELD SATISFYING THE LOCAL MÖBIUS EQUATION

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Abstract

In this paper we prove some results related to a certain vector field satisfying the local Möbius equation on vector fields.

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1. Introduction

A vector field $Z$ on a Riemannian manifold $(M, g)$ is said to satisfy the local Möbius equation if

$$(\nabla^2 Z)(X, Y) = g(\Delta Z, X)Y$$

for all vector fields $X, Y$.

It is known that the existence of solutions $Z$ to the local Möbius equation is related to the conformal structure of the manifold, since the divergence $\text{div} Z$ is a solution of the local Möbius equation, i.e.

$$\text{Hess}_{\text{div}} Z = \frac{\nabla \text{div} Z}{n} \text{Id}$$

and moreover, in such cases $\nabla \text{div} Z$ is a conformal vector field, since $L_{\nabla \text{div} Z} = 2 \text{Hess}_{\text{div}} Z$.

(See also the first four in references.)

The purpose of this paper is to point out such a connection by considering the vector field $Z$ itself. We prove the following:

(Theorem 3.4). A nonzero solution $Z$ of the local Möbius equation is conformal, provided that $M$ is compact.

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