COMPARISON CRITERIA FOR THE OSCILLATION OF MIXED-TYPE IMPULSIVE DIFFERENCE EQUATIONS WITH CONTINUOUS ARGUMENTS

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Abstract

The main objective of this paper is to present a comparison criteria for the oscillation of solutions to mixed-type impulsive difference equation with continuous arguments, without imposing sign restrictions on the coefficients.

Keywords: Difference equations, Continuous arguments, Impulse effects, Mixed-type, Nonoscillation, Oscillation.

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1. Introduction

This paper is concerned with the oscillatory nature of solutions of the following impulsive difference equation (IDE) with continuous arguments:

(1.1)
$$\begin{cases} \Delta_\rho x(t) + p(t)x(t-\tau) + q(t)x(t+\sigma) = 0 & \text{for } t \in [t_0, \infty)\setminus\{\theta_k\}_{k\in\mathbb{N}} \\ x(\theta_k^+) = \lambda_k x(\theta_k) & \text{for } k \in \mathbb{N}, \end{cases}$$

where $t_0 \in \mathbb{R}$, $\rho \in (0, \infty)$, $p, q \in C([t_0, \infty), \mathbb{R})$, $\tau, \sigma \in [0, \infty)$, $\{\lambda_k\}_{k\in\mathbb{N}} \subset (0, \infty)$ and $\{\theta_k\}_{k\in\mathbb{N}} \subset [t_0, \infty)$ is the increasing unbounded sequence of impulse points. Here, $\Delta_\rho x(t) := x(t+\rho) - x(t)$ for $t \in [t_0, \infty)$ and $x(\theta_k^+)$ denotes the right sided limit of x at the impulse point θ_k for some $k \in \mathbb{N}$, and the left sided limits are defined similarly. It should be noted that all solutions of (1.1) are oscillatory in the absence of a subsequence $\{\theta_{k_\ell}\}_{\ell\in\mathbb{N}}$ such that $\{\lambda_{k_\ell}\}_{\ell\in\mathbb{N}} \subset (-\infty, 0)$, since the solution always changes sign at the impulse points $\{\theta_{k_\ell}\}_{\ell\in\mathbb{N}}$.

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