

# THEORY, METHODS AND APPLICATIONS OF INITIAL TIME DIFFERENCE BOUNDEDNESS CRITERIA AND LAGRANGE STABILITY IN TERMS OF TWO MEASURES FOR NONLINEAR SYSTEMS

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## Abstract

This paper investigates the initial time difference equi-boundedness criteria in terms of two measures, initial time difference boundedness and Lagrange Stability in terms of two measures. These are unified with Lyapunov-like functions to establish a variational comparison result. We support our new results with analytic examples and numerical applications.

**Keywords:** Initial time difference, Perturbed differential systems, Boundedness and Lagrange stability, In terms of two measures, Variational comparison result.

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## 1. Introduction

The problems of modern society are both complex and multidisciplinary [1, 2, 3, 4]. It is now recognized that the concept of Lyapunov function [1, 4, 7, 9] can be employed to investigate various qualitative and quantitative properties of dynamic systems. Lyapunov functions serve as a vehicle to transform a given complicated differential system into a relatively simpler system, and as a result it is enough to study the properties of solutions of the simpler system. The application of Lyapunov's second method in boundedness theory [2, 3, 6, 10, 11] has the advantage of not requiring knowledge of the solutions. In order to unify a variety of known concepts of stability and boundedness, it is beneficial to employ two different measures [3, 6, 8, 10, 11] and obtain criteria in terms of these

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