IMAGES AND PREIMAGES
OF (L, M)-GRILLBASES

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Abstract

We introduce the notion of (L, M)-grillbases, where L is a complete lattice and M is a strictly two-sided, commutative quantale lattice. We define two types of image and preimage of (L, M)-grillbases using the Zadeh image and preimage operators. We study the images and preimages of (L, M)-grillbases induced by mappings. We investigate their properties.

1. Introduction

The convergence theory of grills provides a good tool for interpreting topological structures, and plays an important role in topology [11, 12]. Azad [1], Srivastava and Gupta [16] introduced the notion of L-grill on a complete quasi-monoidal lattice (including GL-monoid [2, 3]). Importance of L-grills can be seen in the papers of Khare and Singh [7, 8], Srivastava and Khare [17, 18, 19], where L-grills are used to study the order structure of various families. The present paper arose as a result of such studies, as it gives a structure closely related to L-grills.

Let L be a complete lattice and $\phi : X \to Y$ a mapping. The Zadeh image and preimage operators $\phi_L^{\to} : L^X \to L^Y$ and $\phi_L^{\leftarrow} : L^X \leftarrow L^Y$ are defined by

$$\phi_L^{\to}(f)(y) = \bigvee\{f(x) \mid \phi(x) = y\}, \quad \phi_L^{\leftarrow}(g) = g \circ \phi.$$

Rodabaugh [13, 14, 15] gives two different proofs for all cqml's (complete lattices) L vindicating Zadeh's definitions, first, using the AFT (adjoint functor theorem) to lift the Zadeh operators from traditional operators, and second, classes of naturality diagrams indexed by L to generate Zadeh operators directly from the original mapping.

In this paper, we define (L, M)-grillbases, where L is a complete lattice and M is a strictly two-sided, commutative quantale lattice. We consider the Zadeh image operator

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