AN IMPROVED BAR LEV, BOBOVITCH AND BOUKAI RANDOMIZED RESPONSE MODEL USING MOMENTS RATIOS OF SCRAMBLING VARIABLE

Housila P. Singh*†and Tanveer A. Tarray‡

Abstract

In this paper, we have suggested a new randomized response model and its properties have been studied. The proposed model is found to be more efficient than the randomized response models studied by Bar Lev et al. (2004) and Eichhorn and Hayre (1983). The relative efficiency of the proposed model has been studied with respect to the Bar Lev et al.s (2004) and Eichhorn and Hayres (1983) models. Numerical illustrations are also given to support the present study.
Keywords: Randomized response sampling, Estimation of mean, Respondents protection, Sensitive quantitative variable.

1 Introduction

Warner (1965) introduced a randomized response (RR) model to estimate a population proportion for sensitive attribute such as homosexuality, drug addiction or induced abortion. Greenberg et al. (1971) further made an extension of RR technique for quantitative variables. The RR technique has spawned a vast literature which has been reviewed by Fox and Tracy (1986), Chaudhuri and Mukerjee (1988) and scheers (1992). Some more developments are: Kerkvliet (1994), Gupta and Thornton (2002), Singh and Mathur (2005), Bar Lev et al. (2005), Odumade and Singh (2009), Chaudhuri and Christofides (2013), Singh and Tarray (2013, 2014, 2015), Mashail et al (2015), Tarray and Singh (2015) and Tarray et al. (2015) etc. Eichhorn and Hayre (1983) suggested a multiplicative model to collect information on sensitive quantitative variables like income, tax evasion, amount of drug used etc. For more examples, the reader is referred to Ahsanullah and Eichhorn (1988). According to Eichhorn and Hayre (1983), each respondent in the

*School of Studies in Statistics, vikram University Ujjain - M.P. - India-456010,
†Corresponding Author: Tanveer A. Tarray
‡Department of Computer Science and Engineering, Islamic University of Science and Technology Awantipora Pulwama Kashmir India 192122, Email: tanveerstat@gmail.com

sample is requested to report the scrambled response $Z_i = SY_i$, where Y_i is the real value of the sensitive quantitative variable, and S is the scrambling variable whose distribution is assumed to be known. In other words $E(S) = \theta$ and$V(S) = \gamma^2$ are assumed to be known and positive, where E and V denote the expected value and variance over the randomization device. Then an estimator of the population mean μ_y under the simple random sampling with replacement (SRSWR) due to Eichhorn and Hayre (1983) is given by:

$$\hat{\mu}_{Y(EH)} = \frac{1}{n}\sum_{i=1}^{n}\frac{Z_i}{\theta} \tag{1}$$

with variance

$$V(\hat{\mu}_{Y(EH)}) = \frac{\mu_Y^2}{n}[C_y^2 + C_\gamma^2(1 + C_y^2)], \tag{2}$$

where $C_\gamma^2 = \dfrac{\gamma^2}{\theta^2}$ and $C_y = \dfrac{\sigma_y}{\mu_Y}$.We shall now discuss a randomized response model studied by Bar Lev et al. (2004), say the BBB model. The distribution of the responses is given by:

$$Z_i = \left| \begin{array}{c} Y_iS \text{ with probability } (1-P) \\ Y_i \text{ with probability } P. \end{array} \right. \tag{3}$$

In other words, each respondent is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then he/she is requested to report the real response on the sensitive variable, say Y_i; and if the spinner stops in the non shaded area, then the respondent is required to report the scrambled response, say Y_iS, where S is the scrambled variable. Let P be the radial non shaded area of the spinner as shown in Figure 1.
An unbiased estimator of the population mean Y is given by:

$$\hat{\mu}_{Y(BBB)} = \frac{1}{n[(1-P)\theta + P]}\sum_{i=1}^{n} Z_i \tag{4}$$

with variance under SRSWR sampling given by

$$V[\hat{\mu}_{Y(BBB)}] = \frac{\mu_Y^2}{n}[C_y^2 + (1 + C_y^2)C_P^2], \tag{5}$$

where

$$C_P^2 = \frac{(1-P)\theta^2(1 + C_\gamma^2) + P}{[(1-P)\theta + P]^2} - 1. \tag{6}$$

When the coefficient of variation C_y of the study variable is known, Searls (1964) was the first to consider the problem of estimating the population mean μ_y in the absence of scrambled responses. Later on, with known coefficient of variation C_Y of the study variable Y various authors including Khan (1967), Govindarajulu and Sahai (1972), Gleser and Healy (1976), Sen (1979),

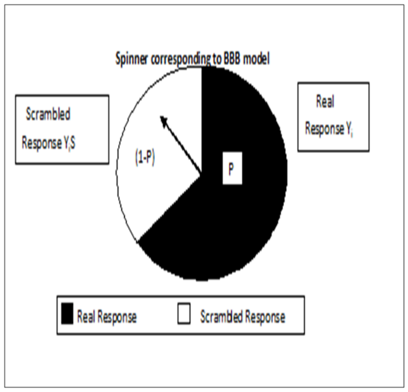

Figure 1: **Bar - lev, Bobovitch and Boukai (2004; BBB) randomized response device**

Tripathi et al. (1983) and Singh and Katiyar (1988) have considered the problem of estimating the population mean μ_Y of the study variable Y. Sen (1978) was first to use the moments ratios of the study variable Y in estimating the population mean μ_Y . Upadhyaya and Singh (1984) have considered the problem of estimating the population mean μ_Y using moments ratios. Singh and Mathur (2005) and Hussain et al. (2013) have used the coefficient of variation C_Y of the study variable Y at the estimation stage in presence of scrambled responses. Singh and Chen (2009) have used the higher order moments of the scrambling variable at the estimation stage for estimating the proportion of a potentially sensitive attribute in survey sampling.
In this paper we have suggested a new randomized response model and its properties are studied. It has been shown that the resulting (optimum) randomized response model depends on the moments ratios such as C_γ (coefficient of variation), $\beta_{1(S)}$ (coefficient of skewness) and $\beta_{2(S)}$ (coefficient of kurtosis) of the scrambling variable S. We have proved the superiority of the proposed randomized response model over Eichhorn and Hayre (1983) and Bar Lev et al. (2004) randomized response models both theoretically and empirically.

2 Suggested Randomized Response model

In the proposed randomized response model, we request an individual to rotate a spinner as shown in Figure 2.
In the proposed randomized response model, the distribution of the response is given by

$$Z_i = \left| \begin{array}{c} Y_i[(1-k)S + K\theta(\dfrac{S-\theta}{\gamma})^2] \text{ with probability } (1-P) \\ Y_i \text{ with probability } P \end{array} \right.$$

.

$$Z_i = \left| \begin{array}{c} Y_i[(1-k)S + K\theta S^{*2}] \text{ with probability } (1-P) \\ Y_i \text{ with probability } P. \end{array} \right. \tag{7}$$

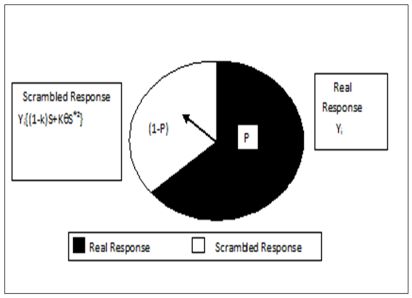

Figure 2: **Spinner of the proposed randomized response model**

The reported response Z_i can also be expressed as
where k is assumed known constant [see Odumade and Singh (2009)] and $S^* = \dfrac{(S-\theta)}{\gamma}$ is the standardized scrambling variable.
In other words, each respondent is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then the respondent is requested to report the real response on the sensitive variable, say Y_i; and if the spinner stops in the non shaded area, then the respondent is required to report the scrambled response, say $Y_i[(1-k)S + K\theta S^{*2}]$. Let P be the proportion of the shaded area of the spinner and $(1-P)$ be the non shaded area of the spinner as shown in Figure 2.
For estimating the population mean μ_Y of the real response on the sensitive quantitative variable Y, a simple random and with replacement sample (SRSWR) of n respondents is selected from the population. Then , we have the following theorem.
Theorem 2.1 An unbiased estimator of the population mean μ_Y is given by

$$\hat{\mu}_Y = \frac{\bar{Z}}{[(1-P)\theta + P]} \tag{8}$$

Proof- We have from (7),
$E(Z_i) = \mu_Y[(1-P)\theta + P]$
Hence, the proposed estimator for μ_Y , based on a random sample of the randomized response; $Z_1, Z_2, ..., Z_n$ is $\hat{\mu}_{Y(HT)} = \dfrac{\bar{Z}}{[(1-P)\theta + P]}$ is unbiased estimator of the population mean μ_Y. Thus the theorem is proved.
The variance of the proposed estimator $\hat{\mu}_{Y(HT)}$ is given in the following theorem.
Theorem 2.2 The variance of $\hat{\mu}_{Y(HT)}$ is given by

$$V(\hat{\mu}_{Y(HT)}) = \frac{\mu_Y^2}{n}[C_y^2+(1+C_y^2)[C_P^2+\frac{(1-P)\theta^2[k^2(\Delta(S)+(\sqrt{\beta_1(S)}-C_\gamma)^2)-2kC_\gamma(C_\gamma-\sqrt{\beta_1(S)})]}{[P+\theta(1-P)]^2}] \tag{9}$$

where $\Delta(S) = [\beta_2(S) - \beta_1(S) - 1], \beta_2(S) = \frac{\mu_4(S)}{\gamma^4}, \beta_1(S) = \frac{\mu_3^2(S)}{\gamma^6}$,
$\mu_3(S) = E(S-\theta)^3$ and $\mu_4(S) = E(S-\theta)^4$.

Proof-

$$V(\hat{\mu}_{Y(HT)}) = V(\bar{Z}) = \frac{V(Z_i)}{n[P+\theta(1-P)]^2} \tag{10}$$

The variance of Z_i is obtained as follows:
$V(Z_i)$=$E(Z_i^2) - (E(Z_i))^2$

$$\left.\begin{aligned} &=(1-p)E[(1-k)^2S^2+\theta^2k^2S^{*4}+2k(1-k)\theta SS^{*2}]E(Y_i^2)+PE(Y_i^2)-(E(Z_i))^2\\ &=\mu_Y^2[(1+C_y^2)[P+(1-P)\theta^2(1+C_\gamma^2)]+(1+C_y^2)\theta^2\\ &\quad(1-P)[k^2(\beta_2(S)-2C_\gamma\sqrt{\beta_1(S)}+C_\gamma^2-1)-2kC_\gamma(C_\gamma-\sqrt{\beta_1(S)})-(P+\theta(1-P))^2]\end{aligned}\right\}$$

Thus, the variance of $\hat{\mu}_{Y(HT)}$is given by

$$\begin{aligned} V(\hat{\mu}_{Y(HT)}) =& \frac{\mu_Y^2}{n}[C_y^2+(1+C_y^2)[C_P^2\\ &+\frac{(1-P)\theta^2[k^2[\Delta(S)+(C_\gamma-\sqrt{\beta_1(S)})^2]-2kC_\gamma(C_\gamma-\sqrt{\beta_1(S)})]}{[P+\theta(1-P)]^2}]\end{aligned}$$

which proves the theorem.

Theorem 2.3 The optimum value of k and the minimum variance of $\hat{\mu}_{Y(HT)}$ are respectively given by

$$k_{opt} = \frac{C_\gamma(C_\gamma-\sqrt{\beta_1(S)})}{[\Delta(S)+(C_\gamma-\sqrt{\beta_1(S)})^2]} \tag{11}$$

and

$$min.V(\hat{\mu}_{Y(HT)}) = \frac{\mu_Y^2}{n}[C_y^2+(1+C_y^2)C_P^2-\frac{(1+C_y^2)\theta^2(1-P)C_\gamma^2(C_\gamma-\sqrt{\beta_1(S)})^2}{[\Delta(S)+(C_\gamma-\sqrt{\beta_1(S)})^2](P+\theta(1-P))^2}] \tag{12}$$

$$min.V(\hat{\mu}_{Y(HT)}) = V(\hat{\mu}_{Y(BBB)}) - \frac{\mu_Y^2(1+C_y^2)\theta^2(1-P)C_\gamma^2(C_\gamma-\sqrt{\beta_1(S)})^2}{n[\Delta(S)+(C_\gamma-\sqrt{\beta_1(S)})^2](P+\theta(1-P))^2} \tag{13}$$

where $V(\hat{\mu}_{Y(BBB)})$ is given by (5).

proof - Differentiating (9) with respect to k and equating to zero, we get the optimum value of k as

$$k_{opt} = \frac{C_\gamma(C_\gamma-\sqrt{\beta_1(S)})}{[\Delta(S)+(C_\gamma-\sqrt{\beta_1(S)})^2]}$$

Substitution of k_{opt} in (9) yields the minimum variance of $\hat{\mu}_{Y(HT)}$ as given in (12) (or (13)).

This completes the proof of the theorem.
Now substituting the value of k_{opt} in place of k in (7) we get the distribution of the responses as

$$Z_{oi} = \left| \begin{array}{c} Y_i[(1-k_{opt})S + K_{opt}\theta S^{*2}] \text{ with probability } (1-P) \\ \\ Y_i \text{ with probability } P. \end{array} \right. \tag{14}$$

Taking expectation of(14), we have
$E(Z_{oi}) = \mu_Y[P + \theta(1-P)]$.
Thus the unbiased estimator of the population mean μ_y based on Z_{oi} is given by

$$\hat{\mu}_{Y(HTO)} = \frac{\bar{Z}_o}{[(1-P)\theta + P]} = \frac{\sum_{i=1}^{n} \frac{\bar{Z}_{oi}}{n}}{[(1-P)\theta + P]} \tag{15}$$

it can be easily shown that the variance of $\hat{\mu}_{Y(HTO)}$ is:

$$V(\hat{\mu}_{Y(HTO)}) = min.V(\hat{\mu}_{Y(HT)}) \tag{16}$$

where $min.V(\hat{\mu}_{Y(HT)})$ is given by (12) (or(13)).
It is well known that $\beta_2(S) > \beta_1(S)+1$ [Kendal and Stuart (1969)]. Hence the optimum estimator $\hat{\mu}_{Y(HTO)}$ is always more efficient than the Bar Lev et al.s (2004) estimator $\hat{\mu}_{Y(BBB)}$ except for population with $\sqrt{\beta_1(S)} = C_\gamma$ for which $\hat{\mu}_{Y(HTO)}$ is as efficient as $\hat{\mu}_{Y(BBB)}$.

3 Efficiency Comparison

(i) Comparison of the proposed optimum estimator $\hat{\mu}_{Y(HTO)}$ (i.e. when the scalar k coincides exactly with that of optimum value k_{opt} of the scalar k) with Bar Lev et al.s (2004) estimator$\hat{\mu}_{Y(BBB)}$.
From (5) and (13), we have

$$V(\hat{\mu}_{Y(BBB)}) - min.V(\hat{\mu}_{Y(HT)})[= V(\hat{\mu}_{Y(HTO)})] = \frac{\mu_Y^2(1+C_y^2)\theta^2(1-P)C_\gamma^2(C_\gamma - \sqrt{\beta_1(S)})^2}{n[\Delta(S) + (C_\gamma - \sqrt{\beta_1(S)})^2](P + \theta(1-P))^2} > 0 \tag{17}$$

which clearly shows that the proposed optimum estimator $\hat{\mu}_{Y(HTO)}$ is better than the estimator$\hat{\mu}_{Y(BBB)}$ due to Bar Lev et al. (2004).
Bar Lev et al. (2004) have proved that for all $P\varepsilon(0,1)$:

$$V(\hat{\mu}_Y = \bar{Y}) < V(\hat{\mu}_{Y(BBB)}) < V(\hat{\mu}_{Y(EH)}). \tag{18}$$

If the distribution of scrambling variables S satisfies

$$0 < \theta < \frac{2\theta^2(1+C_\gamma^2)}{[1+\theta^2(1+C_\gamma^2)]} \tag{19}$$

where

$$\hat{\mu}_Y = \bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}.$$

Thus, under the condition (19) and (17) we have the following inequality:

$$V(\hat{\mu}_{Y(HTO)}) < V(\hat{\mu}_{Y(BBB)}) < V(\hat{\mu}_{Y(EH)}). \tag{20}$$

It follows from (20) that the proposed optimum estimator $\hat{\mu}_{Y(HTO)}$ is more efficient than the Eichhorn and Hayre (1983) estimator $\hat{\mu}_{Y(EH)}$ as long as the condition (19) is satisfied.

(ii) Comparison of the proposed estimator $\hat{\mu}_{Y(HT)}$ with Bar Lev et al.s (2004) estimator$\hat{\mu}_{Y(BBB)}$ when the value of k does not coincide exactly with its optimum value k_{opt} in (11).

From (5) and (9), we have

$$V(\hat{\mu}_{Y(HT)}) = V(\hat{\mu}_{Y(BBB)}) + \frac{\mu_Y^2(1+C_y^2)\theta^2(1-P)}{n(P+\theta(1-P))^2}[k^2A - 2kB], \tag{21}$$

where
$A = [\Delta(S) + (\sqrt{\beta_1(S)} - C\gamma)^2]$ and $B = C_\gamma(C_\gamma - \sqrt{\beta_1(S)})$
We note that

$$V(\hat{\mu}_{Y(HT)}) - V(\hat{\mu}_{Y(BBB)}) = \frac{\mu_Y^2(1+C_y^2)\theta^2(1-P)}{n(P+\theta(1-P))^2}[k^2A - 2kB]$$

which is negative if

$$k^2A - 2kB < 0$$

i.e. if

$$|k - k_{opt}| < |k_{opt}| \tag{22}$$

i.e. if

$$either 0 < k < 2k_{opt} or 2k_{opt} < k < 0 \tag{23}$$

or equivalently,

$$min.(0, 2k_{opt}) < k < max.(0, 2k_{opt}), \tag{24}$$

where $k_{opt} = \frac{B}{A}$

Thus, the proposed estimator proposed estimator $(\hat{\mu}_{Y(HT)})$ is more efficient than Bar Lev et al.s (2004) estimator $(\hat{\mu}_{Y(BBB)})$ as long as the condition (24) is satisfied.

Now, in the following sections we shall discuss our general results in the context of normal and waiting time distributions.

4 Normal Distribution

Let the scrambling variable S have a normal distribution with mean θ and variance γ^2 i.e.$S \sim N(\theta, \gamma^2)$. For this distribution $\sqrt{\beta_1(S)} = 0$ and $\sqrt{\beta_2(S)} = 3 \Rightarrow \Delta(S) = 2$.Thus the optimum value of k_{opt} in (11) and the minimum variance (or the variance of the optimum estimator $\hat{\mu}_{Y(HTO)}$ in (12)(or(13)) respectively reduce to:

$$k_{opt} = \frac{C_\gamma^2}{(2 + C_\gamma^2)} \tag{25}$$

and

$$min.V(\hat{\mu}_{Y(HT)}) = \frac{\mu_y^2}{n}[C_y^2 + (1 + C_y^2)C_P^2 - \frac{(1 + C_y^2)\theta^2(1 - P)C_\gamma^4}{(2 + C_\gamma^2)(P + \theta(1 - P))^2}] = V(\hat{\mu}_{Y(HTO)}) \tag{26}$$

Here $\hat{\mu}_{Y(HTO)}$ is defined by

$$\hat{\mu}_{Y(HTO)} = \frac{\sum_{i=1}^{n} Z_{oi}^*}{n} \tag{27}$$

where Z_{oi}^* is defined by

$$Z_{oi}^* = \left| \begin{array}{c} Y_i[\frac{2S}{(2 + C_\gamma^2)} + \frac{\theta C_\gamma^2 S^{*2}}{(2 + C_\gamma^2)}] \text{ with probability } (1 - P) \\ Y_i \text{ with probability } P. \end{array} \right. \tag{28}$$

It is interesting to note that the optimum $\hat{\mu}_{Y(HTO)}$ in (27) can be used in practice as the coefficient of variation C_γ is known without error.

5 Numerical Illustration using Normal Distribution

To judge the merit of the suggested optimum estimator over Eichhorn and Hayre (1983) estimator $\hat{\mu}_{Y(EH)}$ and the Bar Lev et al. (2004) estimator $\hat{\mu}_{Y(BBB)}$, we have computed the percent relative efficiency (PRE) of the optimum estimator $\hat{\mu}_{Y(HTO)}$ with respect to the estimators $\hat{\mu}_{Y(BBB)}$ and $\hat{\mu}_{Y(EH)}$ by using the formulae:

$$PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(EH)}) = \frac{[C_y^2 + C_\gamma^2(1 + C_y^2)]}{[C_y^2 + C_P^2(1 + C_y^2) - A_1]} \times 100. \tag{29}$$

$$PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(BBB)}) = \frac{[C_y^2 + C_P^2(1 + C_y^2)]}{[C_y^2 + C_P^2(1 + C_y^2) - A_1]} \times 100. \tag{30}$$

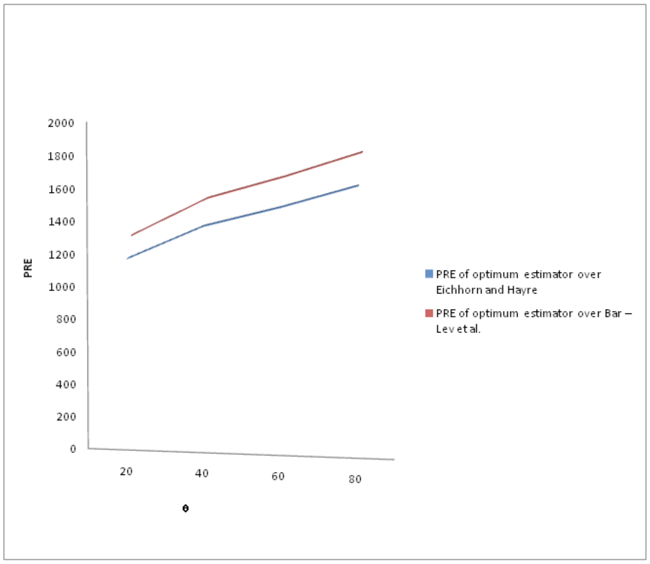

Figure 3: **Graphical representation of the suggested optimum estimator over Eichhorn and Hayre (1983) estimator $\hat{\mu}_{Y(EH)}$ and the Bar Lev et al. (2004) estimator $\hat{\mu}_{Y(BBB)}$.**

for different values of C_y, C_γ, P, θ,
where

$$A_1 = \frac{[(1 + C_y^2)\theta^2(1 - P)C_\gamma^4]}{[2 + C_\gamma^2[P + \theta(1 - P)]^2]} \times 100. \tag{31}$$

Findings are displayed in Tables 1 and 2; and the graphical representation is also given in Figure 3.
Tables 1 and 2 show that the values of $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(EH)})$ and $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(BBB)})$ are much greater than 100. It follows that the proposed optimum estimator $\hat{\mu}_{Y(HTO)}$ is more efficient than Eichhorn and Hayres (1983) estimator $\hat{\mu}_{Y(EH)}$ and Bar Lev et al.s (2004) estimator $\hat{\mu}_{Y(BBB)}$ with considerable gain in efficiency. These facts can be also seen from Figure 3. Thus, based on our numerical results, the use of the proposed estimator $\hat{\mu}_{Y(HTO)}$ is recommended for its use in practice.

6 Waiting Time Distribution

We consider the population, where scrambling variable S follows the waiting time distribution (or distribution of intervals between events in a Poisson process) for which

$$f(s) = 1 - exp(-\frac{s}{\theta}), 0 \leq s \leq \propto, \theta > 0 \tag{32}$$

so that

$$dF(s) = exp(-\frac{s}{\theta})\frac{ds}{\theta} \tag{33}$$

and $E(S) = \theta, V(S) = \theta^2, \mu_3(S) = 2\theta^3$ and $\mu_4(S) = 9\theta^4$
where $C_\gamma = 1$, $\sqrt{\beta_1(S)} = 2, \beta_2(S) = 9, \Delta(S) = 4$. Hence, substituting the values of C_γ, $\sqrt{\beta_1(S)}, \beta_2(S) and \Delta(S) in (11) and (12)$, we have

$$k_{opt} = -\frac{1}{5}, \tag{34}$$

and

$$min.V(\hat{\mu}_{Y(HT)}) = \frac{\mu_y^2}{n}[C_y^2 + (1+C_y^2)C_P^2 - \frac{(1+C_y^2)\theta^2(1-P)}{5(P+\theta(1-P))^2}] = V(\hat{\mu}_{Y(HTO)}) \tag{35}$$

Here the optimum estimator $\hat{\mu}_{Y(HTO)}$ is defined by

$$\hat{\mu}_{Y(HTO)} = \frac{\sum_{i=1}^{n} Z_{oi}^{**}}{n} \tag{36}$$

where Z_{oi}^{**} is defined by

$$Z_{oi}^{**} = \left| \begin{array}{c} Y_i[\frac{6}{5}S - \frac{1}{5}\theta S^{*2}] \text{ with probability } (1-P) \\ \\ Y_i \text{ with probability } P. \end{array} \right. \tag{37}$$

with $S^* = \dfrac{(S-\theta)}{\theta}$.
Thus in this case we note that the optimum estimator$\hat{\mu}_{Y(HTO)}$ in (36) depends on the known quantity θ only.

7 Numerical Illustration using Waiting Time Distribution

To have the tangible idea about the performance of the envisaged optimum estimator $\hat{\mu}_{Y(HTO)}$ over Eichhorn and Hayre (1983) estimator $\hat{\mu}_{Y(EH)}$ and the Bar Lev et al. (2004) estimator $\hat{\mu}_{Y(BBB)}$, we have computed the percent relative efficiency (PRE) of the optimum estimator

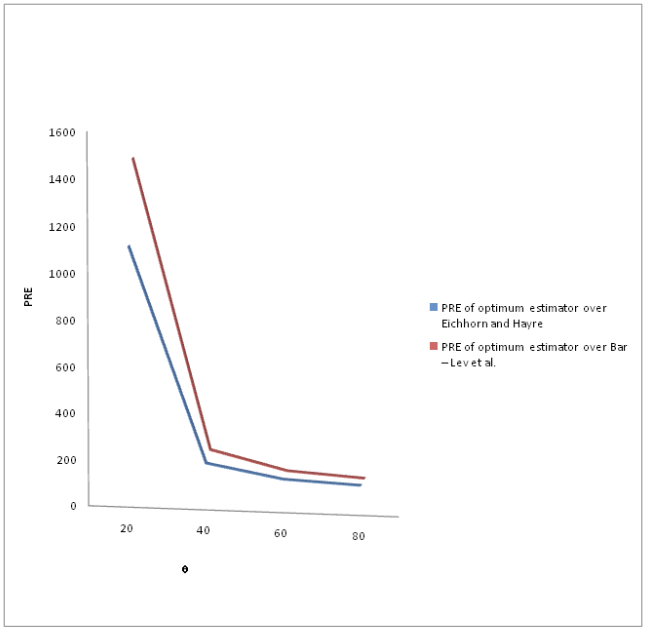

Figure 4: **Graphical representation of the suggested optimum estimator over Eichhorn and Hayre (1983) estimator $\hat{\mu}_{Y(EH)}$ and the Bar Lev et al. (2004) estimator $\hat{\mu}_{Y(BBB)}$.**

$\hat{\mu}_{Y(HTO)}$ with respect to the estimators $\hat{\mu}_{Y(BBB)}$ and $\hat{\mu}_{Y(EH)}$ by using the formulae:

$$PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(EH)}) = \frac{[C_y^2 + C_\gamma^2(1 + C_y^2)]}{[C_y^2 + C_P^2(1 + C_y^2) - A_2]} \times 100. \tag{38}$$

$$PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(BBB)}) = \frac{[C_y^2 + C_P^2(1 + C_y^2)]}{[C_y^2 + C_P^2(1 + C_y^2) - A_2]} \times 100. \tag{39}$$

for different values of C_y, C_γ, P, θ,
where

$$A_2 = \frac{[(1 + C_y^2)\theta^2(1 - P)}{[5[P + \theta(1 - P)]^2]} \times 100. \tag{40}$$

Findings are displayed in Tables 3 and 4; and the graphical representation is also given in Figure 4.

Tables 3 and 4 demonstrate that the values of the percent relative efficiency are greater than 100 for all parameter values tabled. This shows the superiority of the optimum estimator $\hat{\mu}_{Y(HTO)}$ over than Eichhorn and Hayres (1983) estimator $\hat{\mu}_{Y(EH)}$ and Bar Lev et al.s (2004) estimator $\hat{\mu}_{Y(BBB)}$. Graphical representation in Figure 4 also depicts the similar inference. Thus, based on our numerical illustrations, our recommendation is to prefer the proposed estimator $\hat{\mu}_{Y(HTO)}$ in practice.

8 Discussion

In this article, we have suggested a new randomized response model and its properties are studied. It has been shown that the resulting (optimum) randomized response model depends on the moments ratios of the scrambling variable S. We have proved the superiority of the proposed randomized response model over Eichhorn and Hayre (1983) and Bar Lev et al.s (2004) randomized response models both theoretically and empirically.

Acknowledgements

The authors are thankful to the Editor in- Chief - Professor Cem- Kadilar, and to the anonymous learned referees for their valuable suggestions regarding improvement of the paper.

References

[1] Ahsanullah M. and Eichhorn B.H. (1988): On estimation of response from scrambled quantitative data. Pak. Jour. Statist., 4(2), 83-91.

[2] Bar Lev S.K., Bobovitch E. and Boukai B. (2004): A note on Randomized response models for quantitative data. Metrika 60, 225-250.

[3] Chaudhuri A. and Mukerjee R. (1988): Randomized Response: Theory and Techniques. Marcel- Dekker, New York, USA.

[4] Chaudhuri A. and Christofides T. (2013): Indirect Questioning in Sample Surveys. DOI 10.1007/978-3-642-36276-7-3, Springer Verlag Berlin Heidelberg.

[5] Eichhorn B.H. and Hayre, L.S. (1983): Scrambled randomized response methods for obtaining sensitive quantitative data. Jour. Statist. Plann. Infer.,7,307-316.

[6] Fox J.A. and Tracy P.E. (1986): Randomized Response: A method of Sensitive Surveys. Newbury Park, CA: SEGE Publications.

[7] Gjestvang C.R. and Singh S. (2006): A new randomized response model. Jour. Roy. Statist.Soc.68, 523-530.

[8] Gjestvang C.R. and Singh S. (2009): An improved randomized response model: Estimation of mean, Jour. Appl. Statist. 36(12), 1361-1367.

[9] Gleser L.J. and Healy J.D. (1976): Estimating the mean of a normal distribution with known coefficient of variation. Jour. Amer. Statist. Assoc., 71,977-981.

[10] Govindaragulu Z. and Sahai H. (1972): Estimation of the parameters of a normal distribution with known coefficient of variation. Rep. Statist. App. Res. JUSU, 19,86-98.

[11] Gupta S. and Thornton B. (2002): Circumventing social desirability response bias in personal interview surveys. Ameri. Jour. Math. Manag. Sci., 22, 369-383.

[12] Hussain Z., Hamraz M. and shabbier J. (2013) : On alternative estimation technique for randomized response model. Pak. Jour. Statist., 29(3), 283-306.

[13] Khan RA (1968): A note on estimating the mean of a normal distribution with known coefficient of variation. Jour. Amer. Statist. Assoc., 63,1039-1041.

[14] Kerkvliet J.(1994): Estimating a logit model with randomized data: the case of cocaine use. Aust. Jour. Statist.,36,9-20.

[15] Mangat N.S. and Singh R. (1990): An alternative randomized procedure. Biometrika, 77, 439-442.

[16] Mashail, A.M., Zawar H., Bander A., Singh H.P. and Tarray T.A.(2015): Improved randomized response approaches for additive scrambling models . Caspian Journal of Applied Sciences Research. (In Press).

[17] Odumade O. and Singh S. (2009): Improved Bar lev, Bobovitch and Boukai randomized response models. Comm. Statist. Theo. Metho., 38(3), 473-502.

[18] Searls D. T. (1964): The utilization of a known coefficient of variation in the estimation procedure. Jour. Amer. Statist. Assoc.: 59, 12225-12226.

[19] Sen A.R. (1978): Estimation of the mean when the coefficient of variation is known. Comm. Statist. Theor. Method., 7(7), 657-672.

[20] Sen A.R. (1979): Relative efficiency of the estimators of the mean of a normal distribution when coefficient of variation is known. Biometrical. Jour.,21(2), 131-137.

[21] Singh S. and Cheng S.C. (2009): Utilization of higher order moments of scrambling variables in randomized response sampling. Jour. Statist. Plann. Infer.,139, 3377-3380.

[22] Singh H.P. and Katiyar N.P. (1988): A generalized class of estimators for common parameters of two normal distributions with known coefficient of variation. Jour.Ind. Soc. Agri. Statist., 40, (2), 127-149.

[23] Singh H.P. and Mathur N. (2005): Estimation of population mean when coefficient of variation is known using scrambled response technique. Jour. Statist. Plann. Infer., 131 (1), 135-144.

[24] Singh H.P. and Tarray T.A. (2013): A modified survey technique for estimating the proportion and sensitivity in a dichotomous finite population. Int. Jour. Adv. Sci. and Tech. Res., 3(6), 459 472.

[25] Singh H.P. and Tarray T.A. (2014): An alternative to stratified Kim and Wardes randomized response model using optimal (Neyman) allocation. Model Assist. Statist. Appl., 9, 37-62.

[26] Singh H.P. and Tarray T.A. (2015): An efficient use of moments ratios of scrambling variables in randomized response sampling. Communication in Statistics Theory- Methods. (In Press).

[27] Tarray T.A. and Singh H.P. and (2015): An improved new additive model. Gazi University Journal of Sciences (In Press).

[28] Tarray T.A., Singh H.P. and Zaizai Y. (2015): A stratified optional randomized response model. Sociological Methods and Research.doi,10.1177/0049124115605332, 1-15.

[29] Tripathi T.P., Maiti P. and Sharma S.D. (1983): Use of prior information on some parameters in estimating population mean . Sankhya, 45, A, 3, 372-376.

[30] Upadhyaya L.N. and Singh H.P. (1984) : On the estimation of the population mean with known coefficient variation. Biometrical Journal, 26, (8), 915-922.

[31] Warner S.L. (1965): Randomized response: A survey technique for eliminating evasive answer bias. Jour. Amer. Statist. Assoc., 60, 63-69.

Table 1: The $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(EH)})$

θ	P	C_γ	C_y	PRE
20.00	0.10	5.00	0.10	1166.50
40.00	0.10	5.55	0.25	1378.32
60.00	0.10	6.00	0.50	1505.49
80.00	0.10	6.50	0.75	1649.02
20.00	0.20	5.00	0.10	1005.22
40.00	0.20	5.55	0.25	1181.50
60.00	0.20	6.00	0.50	1298.17
80.00	0.20	6.50	0.75	1432.54
20.00	0.30	5.00	0.10	857.59
40.00	0.30	5.55	0.25	1000.69
60.00	0.30	6.00	0.50	1104.65
80.00	0.30	6.50	0.75	1227.07
20.00	0.40	5.00	0.10	722.02
40.00	0.40	5.55	0.25	834.03
60.00	0.40	6.00	0.50	923.61
80.00	0.40	6.50	0.75	1031.80
20.00	0.50	5.00	0.10	597.20
40.00	0.50	5.55	0.25	679.95
60.00	0.50	6.00	0.50	753.89
80.00	0.50	6.50	0.75	845.99
20.00	0.60	5.00	0.10	482.12
40.00	0.60	5.55	0.25	537.15
60.00	0.60	6.00	0.50	594.50
80.00	0.60	6.50	0.75	668.99
20.00	0.70	5.00	0.10	376.14
40.00	0.70	5.55	0.25	404.56
60.00	0.70	6.00	0.50	444.59
80.00	0.70	6.50	0.75	500.24
20.00	0.80	5.00	0.10	279.54
40.00	0.80	5.55	0.25	281.53
60.00	0.80	6.00	0.50	303.54
80.00	0.80	6.50	0.75	339.30

Table 2: The $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(BBB)})$

θ	P	C_γ	C_y	PRE
20.00	0.10	5.00	0.10	1286.41
40.00	0.10	5.55	0.25	1527.41
60.00	0.10	6.00	0.50	1670.15
80.00	0.10	6.50	0.75	1829.85
20.00	0.20	5.00	0.10	1234.45
40.00	0.20	5.55	0.25	1467.05
60.00	0.20	6.00	0.50	1616.15
80.00	0.20	6.50	0.75	1784.75
20.00	0.30	5.00	0.10	1186.85
40.00	0.30	5.55	0.25	1411.58
60.00	0.30	6.00	0.50	1565.74
80.00	0.30	6.50	0.75	1714.95
20.00	0.40	5.00	0.10	11.43.09
40.00	0.40	5.55	0.25	1360.44
60.00	0.40	6.00	0.50	1518.57
80.00	0.40	6.50	0.75	1701.27
20.00	0.50	5.00	0.10	1102.71
40.00	0.50	5.55	0.25	1313.14
60.00	0.50	6.00	0.50	1474.34
80.00	0.50	6.50	0.75	1662.55
20.00	0.60	5.00	0.10	1065.34
40.00	0.60	5.55	0.25	1269.25
60.00	0.60	6.00	0.50	1432.78
80.00	0.60	6.50	0.75	1625.65
20.00	0.70	5.00	0.10	1030.65
40.00	0.70	5.55	0.25	1228.42
60.00	0.70	6.00	0.50	1393.65
80.00	0.70	6.50	0.75	1590.44
20.00	0.80	5.00	0.10	998.36
40.00	0.80	5.55	0.25	1190.34
60.00	0.80	6.00	0.50	1356.73
80.00	0.80	6.50	0.75	1556.80

Table 3: The $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(EH)})$

θ	P	C_γ	C_y	PRE
20.00	0.05	0.40	0.10	1112.83
40.00	0.05	0.50	0.25	191.56
60.00	0.05	0.60	0.50	133.67
80.00	0.05	0.70	0.75	118.70
20.00	0.06	0.40	0.10	683.47
40.00	0.06	0.50	0.25	179.20
60.00	0.06	0.60	0.50	129.80
80.00	0.06	0.70	0.75	116.41
20.00	0.07	0.40	0.10	490.56
40.00	0.07	0.50	0.25	168.14
60.00	0.07	0.60	0.50	126.07
80.00	0.07	0.70	0.75	114.17
20.00	0.08	0.40	0.10	380.96
40.00	0.08	0.50	0.25	158.18
60.00	0.08	0.60	0.50	122.47
80.00	0.08	0.70	0.75	111.97
20.00	0.09	0.40	0.10	310.27
40.00	0.09	0.50	0.25	149.15
60.00	0.09	0.60	0.50	119.01
80.00	0.09	0.70	0.75	109.80
20.00	0.10	0.40	0.10	260.91
40.00	0.10	0.50	0.25	140.94
60.00	0.10	0.60	0.50	115.66
80.00	0.10	0.70	0.75	107.68
20.00	0.11	0.40	0.10	224.48
40.00	0.11	0.50	0.25	133.44
60.00	0.11	0.60	0.50	112.44
80.00	0.11	0.70	0.75	105.59
20.00	0.12	0.40	0.10	196.49
40.00	0.12	0.50	0.25	126.56
60.00	0.12	0.60	0.50	109.32
80.00	0.12	0.70	0.75	103.54

Table 4: The $PRE(\hat{\mu}_{Y(HTO)}, \hat{\mu}_{Y(BBB)})$

θ	P	C_γ	C_y	PRE
20.00	0.05	0.40	0.10	1471.69
40.00	0.05	0.50	0.25	230.24
60.00	0.05	0.60	0.50	150.17
80.00	0.05	0.70	0.75	129.36
20.00	0.06	0.40	0.10	950.47
40.00	0.06	0.50	0.25	223.07
60.00	0.06	0.60	0.50	149.21
80.00	0.06	0.70	0.75	129.09
20.00	0.07	0.40	0.10	716.29
40.00	0.07	0.50	0.25	216.65
60.00	0.07	0.60	0.50	148.29
80.00	0.07	0.70	0.75	128.83
20.00	0.08	0.40	0.10	583.23
40.00	0.08	0.50	0.25	210.86
60.00	0.08	0.60	0.50	147.41
80.00	0.08	0.70	0.75	128.57
20.00	0.09	0.40	0.10	497.42
40.00	0.09	0.50	0.25	205.62
60.00	0.09	0.60	0.50	146.55
80.00	0.09	0.70	0.75	128.32
20.00	0.10	0.40	0.10	437.49
40.00	0.10	0.50	0.25	200.86
60.00	0.10	0.60	0.50	145.73
80.00	0.10	0.70	0.75	128.07
20.00	0.11	0.40	0.10	393.27
40.00	0.11	0.50	0.25	196.50
60.00	0.11	0.60	0.50	144.93
80.00	0.11	0.70	0.75	127.83
20.00	0.12	0.40	0.10	359.30
40.00	0.12	0.50	0.25	192.51
60.00	0.12	0.60	0.50	144.17
80.00	0.12	0.70	0.75	127.59