ON LIPSCHITZ-LORENTZ SPACES AND THEIR ZYGMUND CLASSES

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Received 24 : 10 : 2008 : Accepted 04 : 01 : 2010

Abstract

Let $G$ be a metrizable locally compact abelian group. We prove that $(L_1(G), \text{lip}(\alpha, pq))$, $\tilde{\text{lip}}(\alpha, pq)$, $(L_1(G), \text{Lip}(\alpha, pq))$ and $\text{Lip}(\alpha, pq)$ are isometrically isomorphic, where $\text{Lip}(\alpha, pq)$ and $\text{lip}(\alpha, pq)$ denote the Lipschitz-Lorentz spaces defined on $G$. $(L_1(G), A)$ is the space of multipliers from $L_1(G)$ to $A$ and $\tilde{\text{lip}}(\alpha, pq)$ denotes the relative completion of $\text{lip}(\alpha, pq)$. Also, we characterize the space of multipliers from Lorentz spaces to the Lipschitz-Lorentz-Zygmund classes $L\Lambda_\ast(\alpha, pq; G)$ and $L\lambda_\ast(\alpha, pq; G)$.

Keywords: Lorentz spaces, Lipschitz spaces, Zygmund classes, Relative completion, Multipliers, Translation operator.


1. Introduction and preliminaries

Let $G$ be a metrizable locally compact abelian group with Haar measure $\mu$. In [12], Quek and Yap defined the relative completion $\tilde{A}$ for a linear subspace $A$ of the usual Lebesgue spaces $L_p(G)$ for $1 \leq p < \infty$. They proved that $(L_1(G), A) \subset L_1(G)$ if $p = 1$, and $(L_1(G), A)$ is isometrically isomorphic to $\tilde{A}$ if $p > 1$. In the same paper, they defined the subspaces $\text{Lip}(\alpha, p)$ and $\text{lip}(\alpha, p)$ of the Lebesgue spaces, which are named as Lipschitz spaces. They found the relative completion of $\text{lip}(\alpha, p)$, i.e. $\tilde{\text{lip}}(\alpha, p)$, and stated the relationship between $\tilde{\text{lip}}(\alpha, p)$ and $\text{Lip}(\alpha, p)$. In addition to [12], we can find more interesting results concerning multipliers from $L_1(G)$ to Lipschitz spaces in [5-6].

Quek and Yap used a fixed translation operator while constructing the Lipschitz spaces. However, in [13], they gave the definition of a generalized translation operator, and defined the Lipschitz-Zygmund classes. Also, some theorems about multipliers

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