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ON FUNCTION SPACES WITH WAVELET TRANSFORM IN $\mathbf{L}^p_{\boldsymbol{\omega}}\left(\mathbb{R}^d \times \mathbb{R}_+\right)$

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Abstract

Let ω_1 and ω_2 be weight functions on $\mathbb{R}^d$, $\mathbb{R}^d \times \mathbb{R}_+$, respectively. Throughout this paper, we define $D^{p,q}_{\omega_1,\omega_2}\left(\mathbb{R}^d\right)$ to be the vector space of $f \in L^p_{\omega_1}\left(\mathbb{R}^d\right)$ such that the wavelet transform $W_g f$ belongs to $L^q_{\omega_2}\left(\mathbb{R}^d \times \mathbb{R}_+\right)$ for $1 \le p, q < \infty$, where $0 \ne g \in S\left(\mathbb{R}^d\right)$. We endow this space with a sum norm and show that $D^{p,q}_{\omega_1,\omega_2}\left(\mathbb{R}^d\right)$ becomes a Banach space. We discuss inclusion properties, and compact embeddings between these spaces and the dual of $D^{p,q}_{\omega_1,\omega_2}\left(\mathbb{R}^d\right)$. Later we accept that the variable s in the space $D^{p,q}_{\omega_1,\omega_2}\left(\mathbb{R}^d\right)$ is fixed. We denote this space by $\left(D^{p,q}_{\omega_1,\omega_2}\right)_s\left(\mathbb{R}^d\right)$, and show that under suitable conditions $\left(D^{p,q}_{\omega_1,\omega_2}\right)_s\left(\mathbb{R}^d\right)$ is an essential Banach Module over $L^1_{\omega_1}\left(\mathbb{R}^d\right)$. We obtain its approximate identities. At the end of this work we discuss the multipliers from $\left(D^{p,q}_{\omega_1,\omega_2}\right)_s\left(\mathbb{R}^d\right)$ into $L^\infty_{\omega_1^{-1}}\left(\mathbb{R}^d\right)$, and from $L^1_{\omega_1}\left(\mathbb{R}^d\right)$ into $\left(D^{p,q}_{\omega_1,\omega_2}\right)_s\left(\mathbb{R}^d\right)$.

Keywords: Wavelet transform, Essential Banach module, Approximate identity, Compact embedding, Multipliers space.

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