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ON ISOMETRIES OF $\mathbb{R}^2_{\pi n}$

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Abstract

In this work, we introduce a family of distance functions and show that the group of isometries of the plane associated with the induced metrics is the semi-direct product of the Dihedral group D_{2n} and the translation group $T(2)$.

Keywords: Group, Isometry, Distance function, Metric.

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1. Introduction

Isometries can be viewed as the transformations preserving normed vector spaces. Characterizing the isometries will enable our understanding of the geometry of the space, which is useful in the study of approximation problems, optimization problems, etc. Thus this study also stimulates interactions among different areas: group theory, numerical range, error analysis, [1,2,3,4,8,9,13].

We recall that the symmetry group of a regular $2n$-gon is called the dihedral group and denoted by D_{2n}. It has $4n$ elements, namely $2n$ rotations and $2n$ reflections.

The group of isometries of the Euclidean plane with the usual metric is the semi-direct product of the symmetry group of the unit circle, $O(2)$, and the translation group consisting of all translations of the plane, $T(2)$ [5,6,14]. The groups of isometries of the Taxicab and CC-planes, including the symmetry group of the square and a regular octagon, were given in [10] and [7], respectively.

Here, we introduce a family of distances, $d_{\pi n}$, that includes the Taxicab, Chinese-Checker and Isotaxi distances, [7,8,11], as special cases and then show that the group of isometries of the plane with the $d_{\pi n}$-metric is the semi-direct product of D_{2n} and $T(2)$.

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