

SOME NOTES ON DEDEKIND MODULES

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Abstract

In this paper, we give the relation between a finitely generated torsion free Dedekind module and the endomorphism ring of $O(M)M$. In addition it is proved that the endomorphism ring of a finitely generated torsion free Dedekind module M is a Dedekind domain. Also, we give equivalent condition for Dedekind modules, duo modules and uniform modules. Various properties and characterizations of Dedekind modules over integral domains are considered and consequently, necessary and sufficient conditions for an R -module M to be a Dedekind module are given.

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1. Introduction

Throughout this paper all rings are commutative domains with identity and all modules are unitary.

A nonzero ideal I of R is said to be *invertible* if $II^{-1} = R$, where $I^{-1} = \{x \in K : xI \subseteq R\}$. The concept of an invertible submodule was introduced in [7] as a generalization of the concept of an invertible ideal. Let M be an R -module and let $S = R - \{0\}$. Then

$$T = \{t \in S : tm = 0 \text{ for some } m \in M \text{ implies } m = 0\}$$

is a multiplicatively closed subset of R . Let N be a submodule of M and $N' = \{x \in R_T : xN \subseteq M\}$. A submodule N is said to be *invertible* in M , if $N'N = M$, [7]. Note that N' is an R -submodule of R_T with $R \subseteq N^{-1}$. A nonzero R -module M is called *Dedekind* provided that each nonzero submodule of M is invertible.

Let $O(M) = \{x \in K : xM \subseteq M\}$, the *order* of an R -module M in K . Then $O(M)$ is a subring of K with $R \subseteq O(M)$ and M is an $O(M)$ -module.

Let M be any R -module. We denote the ring of R -endomorphisms of M by $\text{End}(M)$.

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