

ON THE VALUES OF SOME GENERALIZED LACUNARY POWER SERIES WITH ALGEBRAIC COEFFICIENTS FOR LIOUVILLE NUMBER ARGUMENTS[†]

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Abstract

In this work, it is shown that under certain conditions, the values of some generalized lacunary power series with algebraic coefficients from a certain algebraic number field K of degree m for Liouville number arguments belong to either the algebraic number field K or $\bigcup_{i=1}^m U_i$ in Mahler's classification of the complex numbers.

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1. Introduction

A power series $F(z) = \sum_{h=0}^{\infty} c_h z^h$ ($c_h \in \mathbb{C}$, $h = 0, 1, 2, \dots$) with a positive radius of convergence, satisfying the following conditions

$$\begin{cases} c_h = 0, & r_n < h < s_n \quad (n = 1, 2, 3, \dots), \\ c_h \neq 0, & h = r_n \quad (n = 1, 2, 3, \dots), \\ c_h \neq 0, & h = s_n \quad (n = 0, 1, 2, \dots), \end{cases}$$

where $\{s_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=1}^{\infty}$ are two infinite sequences of non-negative rational integers with

$$0 = s_0 \leq r_1 < s_1 \leq r_2 < s_2 \leq r_3 < s_3 \leq \dots, \quad \lim_{n \rightarrow \infty} \frac{s_n}{r_n} = \infty,$$

is called a generalized lacunary power series.

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