

SUBORDINATION RESULTS OF MULTIVALENT FUNCTIONS DEFINED BY CONVOLUTION

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Abstract

Using the method of differential subordination, we investigate some properties of certain classes of multivalent functions, which are defined by means of convolution.

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1. Introduction

Let $A_n(p)$ denote the class of functions of the form

$$(1.1) \quad f(z) = z^p + \sum_{k=n}^{\infty} a_{k+p} z^{k+p}, \quad p, n \in \mathbb{N} = \{1, 2, \dots\},$$

which are analytic and p -valent in the unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$. If f and g are analytic functions in U , we say that f is subordinate to g , written $f(z) \prec g(z)$, if there exists a *Schwarz function* w , which (by definition) is analytic in U , with $w(0) = 0$, and $|w(z)| < 1$ for all $z \in U$, such that $f(z) = g(w(z))$, $z \in U$. Furthermore, if the function g is univalent in U , then we have the equivalence (cf., e.g., [18] and [19])

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

For functions f given by (1.1) and $g \in A_n(p)$ given by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_{k+p} z^{k+p},$$

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