

## SUBORDINATION RESULTS OF MULTIVALENT FUNCTIONS DEFINED BY CONVOLUTION

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### Abstract

Using the method of differential subordination, we investigate some properties of certain classes of multivalent functions, which are defined by means of convolution.

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### 1. Introduction

Let  $A_n(p)$  denote the class of functions of the form

$$(1.1) \quad f(z) = z^p + \sum_{k=n}^{\infty} a_{k+p} z^{k+p}, \quad p, n \in \mathbb{N} = \{1, 2, \dots\},$$

which are analytic and  $p$ -valent in the unit disc  $U = \{z \in \mathbb{C} : |z| < 1\}$ . If  $f$  and  $g$  are analytic functions in  $U$ , we say that  $f$  is subordinate to  $g$ , written  $f(z) \prec g(z)$ , if there exists a *Schwarz function*  $w$ , which (by definition) is analytic in  $U$ , with  $w(0) = 0$ , and  $|w(z)| < 1$  for all  $z \in U$ , such that  $f(z) = g(w(z))$ ,  $z \in U$ . Furthermore, if the function  $g$  is univalent in  $U$ , then we have the equivalence (cf., e.g., [18] and [19])

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

For functions  $f$  given by (1.1) and  $g \in A_n(p)$  given by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_{k+p} z^{k+p},$$

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