SOME APPLICATIONS OF FRACTIONAL CALCULUS OPERATORS TO THE ANALYTIC PART OF HARMONIC UNIVALENT FUNCTIONS

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Abstract

Recently, Jahangiri [4] studied the harmonic starlike functions of order \( \alpha \), and he defined the class \( T_{\alpha}(\alpha) \) consisting of functions \( f = h + \bar{g} \), where \( h \) and \( g \) are the analytic and the co-analytic part of the function \( f \), respectively. In [3] the author introduced the class \( T_{\alpha}(\alpha, \beta) \) of analytic functions and he proved various coefficient inequalities and growth and distortion theorems, and obtained the radius of convexity for the function \( h \) if the function \( f \) belongs to the classes \( T_{\alpha}(\alpha) \) and \( T_{\alpha}(\alpha, \beta) \). In this paper, we derive various distortion theorems for the fractional calculus and the fractional integral operator of the function \( h \), the analytic part of the function \( f \), if the function \( f \) belongs to the class \( T_{\alpha}(\alpha, \beta) \).

Keywords: Harmonic, Analytic and univalent functions. Fractional calculus and fractional integral operator.

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1. Introduction and Definitions

A continuous complex valued function \( f = u + iv \) defined in a simply connected complex domain \( \mathcal{D} \) is said to be harmonic in \( \mathcal{D} \) if both \( u \) and \( v \) are real harmonic in \( \mathcal{D} \). In any simply connected domain we can write \( f = h + g \), where \( h \) and \( g \) are analytic in \( \mathcal{D} \). We call \( h \) the analytic part and \( g \) the co-analytic part of \( f \). A necessary and sufficient condition for \( f \) to be locally univalent and sense preserving in \( \mathcal{D} \) is that \( |h'(z)| > |g'(z)| \) in \( \mathcal{D} \).

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