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ON THE GENERALIZED BESSEL HEAT EQUATION RELATED TO THE GENERALIZED BESSEL DIAMOND OPERATOR

Aziz Sağlam*† and Hüseyin Yıldırım*

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Abstract

In this article, we study the equation

$$\frac{\partial}{\partial t}u(x,t) = c^2 \otimes_B^{m,k} u(x,t)$$

with the initial condition $u(x,0) = f(x)$ for $x \in \mathbb{R}_n^+$. Here the operator $\otimes_B^{m,k}$ is called the Generalized Bessel Diamond Operator, iterated k times, and is defined by

$$\otimes_B^{m,k} = \Big[\big(B_{x_1} + B_{x_2} + \cdots + B_{x_p}\big)^m - \big(B_{x_{p+1}} + \cdots + B_{x_{p+q}}\big)^m \Big]^k,$$

where k and m are positive integers, $p+q=n$, $B_{x_i} = \dfrac{\partial^2}{\partial x_i^2} + \dfrac{2v_i}{x_i}\dfrac{\partial}{\partial x_i}$, $2v_i = 2\alpha_i + 1$, $\alpha_i > -\frac{1}{2}$, $x_i > 0$, $i = 1, 2, \dots, n$, n being the dimension of the space $\mathbb{R}_n^+$, $u(x,t)$ is an unknown function of the form $(x,t) = (x_1, \dots, x_n, t) \in \mathbb{R}_n^+ \times (0, \infty)$, $f(x)$ is a given generalized function and c a constant. We obtain the solution of this equation, which is related to the spectrum and the kernel, the so called generalized Bessel diamond heat kernel. Moreover, the generalized Bessel diamond heat kernel is shown to have interesting properties and to be related to the kernel of an extension of the heat equation.

Keywords: Heat kernel, Dirac-delta distribution, Bessel Diamond Operator, Spectrum.

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*Department of Mathematics, Faculty of Science and Arts, Afyon Kocatepe University, Afyon, Turkey. E-mail: (A. Sağlam) azizsaglam@aku.edu.tr (H. Yıldırım) hyildir@aku.edu.tr

†Corresponding Author.