SOME IDENTITIES FOR GENERALIZED FIBONACCI AND LUCAS SEQUENCES

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Abstract

In this study, we define a generalization of Lucas sequence $\{p_n\}$. Then we obtain Binet formula of sequence $\{p_n\}$. Also, we investigate relationships between generalized Fibonacci and Lucas sequences.

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1. Introduction

For $n \geq 2$, the Fibonacci and Lucas numbers are defined by following recurrence relations

$$F_0 = 0, \quad F_1 = 1, \quad F_n = F_{n-1} + F_{n-2}$$

and

$$L_0 = 2, \quad L_1 = 1, \quad L_n = L_{n-1} + L_{n-2}.$$

And Fibonacci and Lucas numbers' Binet formulas are known as,

$$F_n = \frac{\tau^n - \gamma^n}{\tau - \gamma} \qquad \text{and} \qquad L_n = \tau^n + \gamma^n$$

where $n \geq 0$ and τ, γ are roots of $x^2 - x - 1 = 0$.

These sequences have been generalized in many ways. For example, in [1], the author generalized the sequences $\{F_n\}$ and $\{L_n\}$ as follows,

$$W_n = AW_{n-1} + BW_{n-1}, \quad W_0 = a, \quad W_1 = b \quad \text{for} \quad n \geq 2,$$

where a, b, A and B are arbitrary integers.

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