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A COUPLED FIXED POINT RESULT IN PARTIALLY ORDERED PARTIAL METRIC SPACES THROUGH IMPLICIT FUNCTION

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Abstract

In this manuscript, we discuss the existence of coupled fixed points in the context of partially ordered metric spaces through implicit relations for mappings $F : X \times X \to X$ such that F has the mixed monotone property. Our main theorem improves and extends various results in the literature. We also state an example to illustrate our work.

Keywords: Coupled fixed point; Mixed monotone property; Implicit relation; Ordered partial metric space

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1. Introduction and Preliminaries

The notion of partial metric space, introduced by Matthews [46], is a generalization of metric space defined by Fréchet [20] in 1906. Roughly speaking the most remarkable property in a partial metric space is that the self-distance need not be zero. Nonzero self-distance makes perfect sense in the framework of Computer Sciences, in particular, in the Domain Theory and Semantics (see e.g., [39, 40, 51, 23, 57, 65, 66, 74, 75]). In the paper [46], Matthews proved an analog of the well-known Banach contraction principle in the context of complete partial metric spaces. After this result, many authors have conducted further research on fixed point theorems in the same class of spaces. Furthermore they studied topological properties of partial metric spaces (see e.g.,[2]-[7], [9, 16, 18, 24, 25] [35]-[33],[40],[60]-[64],[67, 73]).

A partial metric is a function $p : X \times X \to [0, \infty)$ which satisfies the following conditions
(P1) $p(x, y) = p(y, x)$,
(P2) If $p(x, x) = p(x, y) = p(y, y)$, then $x = y$,

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