NEW INEQUALITIES FOR MAXIMUM MODULUS VALUES OF POLYNOMIAL FUNCTIONS

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Abstract

In this paper we establish new inequalities for the maximum modulus values of polynomial functions which do not have zero as a root, and which have zero as a multiple root on two hyperbolic regions. Similar inequalities for the particular case where the region is a ring are also given.

Keywords: Mathematical analysis, Hyperbolic region, Polynomials functions, Maximum modulus values, Inequality.

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1. Introduction

The Maximum Modulus Theorem [3, 4] states:

Let $U$ be open and connected, $f$ holomorphic on $U$ and nonconstant.

(i) For any $z_0 \in U$, there is $z_1 \in U$ such that $|f(z_0)| < |f(z_1)|$. In other words, $|f|$ cannot attain its maximum on $U$.

(ii) Suppose $U$ is bounded and $f$ is continuous on $\text{cl } U$. Then $|f|$ attain its maximum on the boundary $\partial U$.

(iii) Let $U$ and $f$ be as (ii). Then

$$\text{Sup}\{|f(z)| : z \in \text{cl } U\} = \text{Sup}\{|f(z)| : z \in \partial U\}.$$

Let $f, g : C \to C$ be complex-valued polynomial functions of degrees $m \geq 1$, $n \geq 1$, respectively, of a complex variable $z$, and $M_f = \max_{|z|=R} |f(z)|$, $M_g = \max_{|z|=R} |g(z)|$ and $M_{f,g} = \max_{|z|=R} |f(z),g(z)|$ ($R > 1$). It is shown in [1] that, if $z = 0$ is not a root of the given polynomials,

$$M_{f,g} \geq \delta_1 \cdot M_f \cdot M_g$$

with $\delta_1 = \frac{1}{\sqrt{m}} \cdot \frac{1}{\sqrt{n}}$.

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