FINITE LENGTH MODULES OVER HNPs

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Received 31:01:2008 : Accepted 20:07:2009

Abstract
We characterize a finite length module over HNP that is annihilated by an invertible ideal. We also characterize finite length modules over HNP that have no composition factors annihilated by an invertible ideal. The two characterizations are used to prove Levy’s Theorem about the decomposition of finite length modules over HNPs. We also prove that the ring of matrices over a uniserial ring is serial by generalizing the technique of proving that the ring of matrices over a simple ring is simple. This is done by exploring the form of a one sided ideal of a matrix ring. We also characterize a uniserial Artinian ring as a local, principle ideal, Artinian ring. We use the two results to prove that the component that is annihilated by an invertible ideal in the Levy decomposition is a serial module.

Keywords: HNP ring, Uniserial, Serial, Invertible ideal.

2000 AMS Classification: 16D40, 16D70.

1. Results

The Theorem of decomposing a finite length module over HNPs [4], will be proved in another way. Let $R$ be a HNP ring. First, we state Lemma 4.3(i) in [4].

1.1. Lemma. Let $S, T$ be simple $R$-modules. If there is an invertible ideal annihilating $S$ but not annihilating $T$, then $\text{Ext}^1(R, S, T) = 0$.

In the next Theorem, we characterize a finite length $R$-module that is annihilated by an invertible ideal.

1.2. Theorem. Let $M$ be a finite length $R$-module. Then $M$ is annihilated by an invertible ideal if and only if each of its simple submodules is annihilated by an invertible ideal.

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