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COVERING GROUPOIDS OF CATEGORICAL GROUPS

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Abstract

If X is a topological group, then its fundamental groupoid $\pi_1(X)$ is a group-groupoid which is a group object in the category of groupoids. Further if X is a path connected topological group which has a simply connected cover, then the category of covering groups of X and the category of covering groupoids of $\pi_1(X)$ are equivalent. In this paper we prove that if (X, x_0) is an H-group, then the fundamental groupoid $\pi_1(X)$ is a weak categorical group. This enables one to prove that the category of the covering spaces of an H-group (X, x_0) is equivalent to the category of covering groupoids of the weak categorical group $\pi_1(X)$.

Keywords: H-group, covering groupoid and categorical group.

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Introduction

Covering spaces are studied in algebraic topology, but they have important applications in many other branches of mathematics including differential topology, the theory of topological groups and the theory of Riemann surfaces.

One of the ways of expressing the algebraic content of the theory of covering spaces is using groupoids and the fundamental groupoids. The latter functor gives an equivalence of categories between the category of covering spaces of a reasonably nice space X and the category of covering groupoids of $\pi_1(X)$.

If X is a connected topological group with identity e and $p\colon (\widetilde{X}, \tilde{e}) \to (X, e)$ is a covering map of pointed spaces such that $\widetilde{X}$ is simply connected, then $\widetilde{X}$ becomes a topological group with identity $\tilde{e}$ such that p is a morphism of topological groups (see for example [6, Proposition 5] and [11, Theorem 10.42]).

The problem of universal covers of non-connected topological groups was first studied by Taylor in [12]. He proved that a topological group X determines an obstruction class

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