ON SOLUTIONS OF A GENERALIZED QUADRATIC FUNCTIONAL EQUATION OF PEXIDER TYPE

Mohammad Janfada∗† and Rahele Shourvazi‡

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Abstract

In this paper we study general solutions of the following Pexider functional equation

\[(4 - k)f_1 \left( \sum_{i=1}^{k} x_i \right) + \sum_{j=2}^{k} f_j \left( \left( \sum_{i=1, i \neq j}^{k} x_i \right) - x_j \right) \]
\[+ f_{k+1} \left( -x_1 + \sum_{i=2}^{k} x_i \right) = 4 \sum_{j=1}^{k} f_{k+j+1} (x_j), \]

on a vector space over a field of characteristic different from 2, for \( k \geq 3 \).

Keywords: Quadratic functional equation, Pexiderized quadratic functional equation.


1. Introduction

It is easy to see that the quadratic function \( f(x) = x^2 \) is a solution for each of the following functional equations

\[ f(x + y) + f(x - y) = 2f(x) + 2f(y) \]
\[ f(x + y + z) + f(x - y + z) + f(x + y - z) + f(-x + y + z) \]
\[ = 4f(x) + 4f(y) + 4f(z). \]

So, it is natural that these equations are called quadratic functional equations. In particular, every solution of the original quadratic functional equation (1.1) is said to be a quadratic function.

∗Department of Pure Mathematics, Ferdowsi University of Mashhad, P.O. Box 1159-91775, Mashhad, Iran. E-mail: mjanfada@gmail.com
†Corresponding Author.
‡Faculty of Mathematics and Computer Sciences, Hakim Sabzevarya University, P.O. Box 397, Sabzevar, Iran. E-mail: rshurvarzy@gmail.com