A generalized class of difference type estimators for population median in survey sampling

Javid Shabbir*† and Sat Gupta‡

Abstract
In this paper, we propose a generalized class of difference type estimators of finite population median in simple and stratified random sampling. The expressions for bias and mean square error are derived up to first order of approximation. Numerical comparisons reveal that the proposed class of estimators performs better than the unbiased sample median estimator, ratio estimator, exponential estimator, usual difference estimator, Rao [10] estimator and other difference type estimators.

Keywords: Auxiliary variable, Median, Bias, Mean square error (MSE), efficiency.

2000 AMS Classification: 62D05

Received: 20.04.2015 Accepted: 09.12.2015 Doi: 10.15672/HJMS.201610614759

1. Introduction
Several authors have developed some estimators for the finite population mean under different sampling schemes. However lesser degree of attention has been paid to estimation of median. Kuk and Mak [8] introduced a median estimator that makes use of the auxiliary information. Gupta et al. [5] have suggested a class of estimators for population median using two auxiliary variables. Other important contributions in this area include Al and Cingi [2], Singh and Solanki [14], Jhajj et al. [7], Sharma and Singh [12], Solanki and Singh [15] and Aladag and Cingi [3].

In this paper we consider the problem of median estimation for finite population and propose a generalized class of difference type estimators that makes use of the auxiliary information in simple and stratified random sampling.

Consider a finite population with \( N \) units. Let \( y_i \) and \( x_i \) (\( i = 1, 2, ..., N \)) be the values on the \( i \)th unit for the study variable (\( Y \)) and the auxiliary variable (\( X \)) respectively. Let

*Department of Statistics Quaid-i-Azam University, Islamabad, Pakistan, Email: javidshabbir@gmail.com
†Corresponding Author.
‡Department of Mathematics and Statistics, the University of North Caroline at Greensboro, Greensboro NC 27412, USA, Email: sngupta@uncg.edu.
us draw a sample of size \(n\) from this population by using simple random sampling without replacement. Let \(M_y\) and \(M_x\) respectively be the population medians and \(\hat{M}_y\) and \(\hat{M}_x\) respectively be the sample medians for \(Y\) and \(X\). Let the correlation coefficient between \((\hat{M}_y, \hat{M}_x)\) be \(\rho_{\hat{M}_y\hat{M}_x} = \rho_c = 4P_{11}(y, x) - 1\), where \(P_{11}(y, x) = P(Y \leq \hat{M}_y \cap X \leq \hat{M}_x)\).

It is assumed that the limiting distribution of \((Y, X)\) is a continuous distribution with marginal densities \(f_y(y)\) and \(f_x(x)\) for \(Y\) and \(X\) respectively. It is further assumed that \(f_y(M_y)\) and \(f_x(M_x)\) are positive.

To obtain the properties of the proposed median estimator, we define the following error terms. Let \(e_0 = (\hat{M}_y - M_y)/M_y\) and \(e_1 = (\hat{M}_x - M_x)/M_x\) such that \(E(e_0) = E(e_1) = 0\). To first degree of approximation, we have \(E(e_0^2) = \lambda C_{\hat{M}_y}^2, E(e_1^2) = \lambda C_{\hat{M}_x}^2, E(e_0 e_1) = \lambda C_{\hat{M}_y\hat{M}_x}\), where \(C_{M_y} = 1/[M_y f_y(M_y)]\), \(C_{M_x} = 1/[M_x f_x(M_x)]\), \(C_{\hat{M}_y\hat{M}_x} = \rho_c C_{\hat{M}_y} C_{\hat{M}_x}\) and \(\lambda = \frac{1}{\pi} \left(\frac{1}{\pi} - 1\right)\).

2. Some existing median estimators in simple random sampling

In this section, we discuss some of the existing estimators of population median \((M_y)\). All expressions are given to first degree approximation.

The most common median estimator is the sample median \((\hat{M}_y)\) whose variance is, given by

\[
(2.1) \quad \text{Var}(\hat{M}_y) = \lambda M_y^2 C_{\hat{M}_y}^2 = \text{MSE}(\hat{M}_y).
\]

Kuk and Mak [8] have introduced the following ratio estimator:

\[
(2.2) \quad \hat{M}_{\text{R}} = \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right),
\]

where \(M_x\) is known.

The bias and \(\text{MSE}\) of \(\hat{M}_{\text{R}}\), are given by

\[
(2.3) \quad \text{Bias}(\hat{M}_{\text{R}}) \cong \lambda M_y \left( C_{\hat{M}_y}^2 - C_{M_y\hat{M}_x} \right)
\]

and

\[
(2.4) \quad \text{MSE}(\hat{M}_{\text{R}}) \cong \lambda M_y^2 \left( C_{\hat{M}_y}^2 + C_{\hat{M}_x}^2 - 2C_{M_y\hat{M}_x} \right).
\]

The exponential ratio type estimator is given by

\[
(2.5) \quad \hat{M}_{\text{EX}} = \hat{M}_y \exp \left( \frac{M_x - \hat{M}_x}{\hat{M}_x + \hat{M}_x} \right).
\]

The bias and \(\text{MSE}\) of \(\hat{M}_{\text{EX}}\), are given by

\[
(2.6) \quad \text{Bias}(\hat{M}_{\text{EX}}) \cong \lambda M_y \left( \frac{3}{8} C_{\hat{M}_x}^2 - \frac{1}{2} C_{M_y\hat{M}_x} \right)
\]

and

\[
(2.7) \quad \text{MSE}(\hat{M}_{\text{EX}}) \cong \lambda M_y^2 \left( C_{\hat{M}_y}^2 + \frac{1}{4} C_{\hat{M}_x}^2 - C_{M_y\hat{M}_x} \right).
\]

An unbiased difference estimator \((\hat{M}_D)\), is given by

\[
(2.8) \quad \hat{M}_D = \hat{M}_y + d \left( M_x - \hat{M}_x \right),
\]

where \(d\) is an unknown constant.

The minimum \(\text{MSE}\) of \(\hat{M}_D\), at optimum value of \(d\) i.e. \(d_{\text{opt}} = \frac{M_y \rho_c C_{M_x}}{M_x C_{\hat{M}_x}}\) is given by

\[
(2.9) \quad \text{MSE}(\hat{M}_D)_{\text{min}} = \lambda M_y^2 C_{\hat{M}_y}^2 \left(1 - \rho_c^2\right).
\]
The minimum $MSE$ of $\hat{M}_D$ is always smaller than the sample median estimator ($\hat{M}_y$), ratio estimator ($\hat{M}_R$) and exponential type estimator ($\hat{M}_{EX}$).

Some more difference type estimators $\hat{M}_D(i = 1, 2, 3)$ which are similar to Rao [10], Gupta et al. [5] and Shabbir and Gupta [11] estimators respectively, are given by

$$
\hat{M}_{D1} = d_1 \hat{M}_y + d_2 \left( M_x - \hat{M}_x \right),
$$

$$
\hat{M}_{D2} = \left[ d_3 \hat{M}_y + d_4 (M_x - \hat{M}_x) \right] \left( \frac{M_x}{M_y} \right),
$$

$$
\hat{M}_{D3} = \left[ d_5 \hat{M}_y + d_6 (M_x - \hat{M}_x) \right] \exp \left( \frac{M_x - \hat{M}_x}{M_x + \hat{M}_x} \right),
$$

where $d_i (i = 1, 2, \ldots, 6)$ are unknown constants whose optimal values are to be determined.

The biases and minimum $MSE$s of $\hat{M}_{D1}(i = 1, 2, 3)$, are given by

$$
Bias(\hat{M}_{D1}) \cong M_y (d_1 - 1),
$$

$$
Bias(\hat{M}_{D2}) \cong (d_3 - 1) M_y + d_3 \lambda M_y (C_{Mx}^2 - C_{Myx}) + d_4 M_x \lambda C_{Mx}^2,
$$

$$
Bias(\hat{M}_{D3}) \cong (d_5 - 1) M_y + d_5 \lambda M_y \left[ \frac{3}{8} C_{Mx}^2 - \frac{1}{2} C_{Myx} \right] + \frac{1}{2} d_6 M_x \lambda C_{Mx}^2,
$$

$$
MSE(\hat{M}_{D1})_{\text{min}} \cong M_y \lambda C_{Mx}^2 (1 - \rho_c^2) \left[ 1 + \lambda C_{My}^2 (1 - \rho_c^2) \right],
$$

$$
MSE(\hat{M}_{D2})_{\text{min}} \cong M_y^2 \left[ (1 - \lambda C_{Mx}^2) - \frac{(1 - \lambda C_{Mx}^2)^2}{(1 - \lambda C_{Mx}^2) + \lambda C_{My}^2 (1 - \rho_c^2)} \right]
$$

or

$$
MSE(\hat{M}_{D2})_{\text{min}} \cong M_y^2 \left[ \frac{1 - \lambda C_{Mx}^2}{1 - \lambda C_{Mx}^2 + \lambda C_{My}^2 (1 - \rho_c^2)} \right],
$$

$$
MSE(\hat{M}_{D3})_{\text{min}} \cong M_y^2 \left[ (1 - \frac{1}{4} \lambda C_{Mx}^2) - \frac{(1 - \frac{1}{4} \lambda C_{Mx}^2)^2}{1 + \lambda C_{My}^2 (1 - \rho_c^2)} \right]
$$

or

$$
MSE(\hat{M}_{D3})_{\text{min}} \cong M_y^2 \left[ \frac{\lambda C_{My}^2 (1 - \rho_c^2) - \frac{1}{4} \lambda^2 C_{Mx}^4 - \frac{1}{4} \lambda^2 C_{My}^2 C_{Mx}^2 (1 - \rho_c^2)}{1 + \lambda C_{My}^2 (1 - \rho_c^2)} \right],
$$

where optimum values of $d_i (i = 1, 2, \ldots, 6)$ are given by:

$$
d_{1(\text{opt})} = \frac{1}{1 + \lambda C_{My}^2 (1 - \rho_c^2)} \quad d_{2(\text{opt})} = \frac{M_y}{M_x} \left[ \frac{\rho C_{My} / C_{Mx}}{1 + \lambda C_{My}^2 (1 - \rho_c^2)} \right], \quad d_{3(\text{opt})} = \frac{M_y}{M_x} \left[ \frac{1 - \lambda C_{My}^2 / (1 - \rho_c^2)}{1 + \lambda C_{My}^2 (1 - \rho_c^2)} \right],
$$

$$
d_{4(\text{opt})} = \frac{M_y}{M_x} \left[ 1 + d_{3(\text{opt})} \left( \frac{\rho C_{My} / C_{Mx}}{\rho C_{My} / C_{Mx}} - 2 \right) \right], \quad d_{5(\text{opt})} = \frac{1 - (\rho C_{My} / 8)}{1 + \lambda C_{My}^2 (1 - \rho_c^2)},
$$

$$
d_{6(\text{opt})} = \frac{M_y}{M_x} \left[ \frac{1}{4} + d_{5(\text{opt})} \left( \frac{\rho C_{My} / C_{Mx}}{\rho C_{My} / C_{Mx}} - 1 \right) \right].
$$

We can get the corresponding bias of $\hat{M}_{D1}(i = 1, 2, 3)$ by substituting the optimum values of $d_i (i = 1, 2, \ldots, 6)$ in Eqs (2.13)–(2.15).
3. Proposed median estimator in simple random sampling

Motivated by Singh and Solanki [14], Jhajji et al. [7], Sharma and Singh [12] and Solanki and Singh [15], we propose the following generalized difference type estimator of \( \hat{M}_y \):

\[
\hat{M}^G_y = \left[ m_1 \hat{M}_y + m_2 (M_x - \hat{M}_x) \right] \\
\left( \frac{a M_x + b}{a M_x + b} \right)^{\alpha_1} \exp \left\{ \frac{\alpha_2 a (M_x - \hat{M}_x)}{a \{(\gamma - 1) M_x + M_x\} + 2b} \right\},
\]

where \( a \) and \( b \) are the known population parameters; \( m_1 \) and \( m_2 \) are unknown constants whose values are to be determined and \( \alpha_1 \), \( \alpha_2 \) and \( \gamma \) are scalar quantities which can take different values.

Note: By substituting different values of \( \alpha_1, \alpha_2, \gamma, a, b \), we can obtain many estimators as described earlier.

Let substitute \( \alpha_1 = b = 0, \alpha_2 = \gamma = a = 1 \) in Eq.(3.1), the class of generalized type estimators becomes

\[
\hat{M}^G_{\hat{Y}} = \left[ m_1 \hat{M}_y + m_2 (M_x - \hat{M}_x) \right] \left[ \exp \left( \frac{M_x}{M_x - 1} \right) \right].
\]

Solving Eq.(3.2), the bias and minimum \( \text{MSE} \) of \( \hat{M}^G_{\hat{Y}} \) at optimum values: \( m_1(\text{opt}) = \frac{1 - \frac{1}{2} \lambda C_M^2}{1 + \frac{1}{2} \lambda C_M^2 (1 - \rho^2)} \) and \( m_2(\text{opt}) = \frac{M_y}{M_x} \left[ 1 + m_1(\text{opt}) \right] \left( \frac{C_y C_M}{C_M^2} - 2 \right) \), are given by

\[
\text{Bias}(\hat{M}^G_{\hat{Y}}) \cong (m_1 - 1) M_y + m_2 M_y \lambda \left( \frac{3}{2} C_M^2 - C_M \right) + m_2 M_x \lambda C_M^2
\]

and

\[
\text{MSE}(\hat{M}^G_{\hat{Y}}) \cong M_y^2 \left[ (1 - \lambda C_M^2) - \frac{1 - \lambda C_M^2}{1 + \lambda C_M^2 - \lambda C_M^2 (1 - \rho^2)} \right]
\]

or

\[
\text{MSE}(\hat{M}^G_{\hat{Y}}) \cong M_y^2 \left[ \frac{\lambda C_M^2 (1 - \rho^2) - \frac{1}{2} \lambda C_M^2 (1 - \rho^2) - \frac{1}{2} \lambda C_M^2 (1 - \rho^2)}{1 + \lambda C_M^2 (1 - \rho^2)} \right].
\]

4. Comparison of estimators in simple random sampling

In this section, we compare the mean square error of the new class of generalized difference type estimators \( \hat{M}^G_{\hat{Y}} \) at optimum condition with other existing estimators.

**Condition (i)**

By (2.1) and (3.5), \( \text{MSE}(\hat{M}^G_{\hat{Y}})_\text{min} < \text{MSE}(\hat{M}_y) \) if

\[
\frac{1}{\pi^2} \left[ \lambda C_M^2 \rho_c^2 + \lambda^2 \theta_1 \right] > 0,
\]

where \( \theta_1 = \frac{1}{2} C_M^2 + C_M^2 (1 - \rho^2) + C_M^2 C_M^2 (1 - \rho^2) \) and \( \theta_2 = 1 + \lambda C_M^2 (1 - \rho^2) \).

**Condition (ii)**

By (2.4) and (3.5), \( \text{MSE}(\hat{M}^G_{\hat{Y}})_\text{min} < \text{MSE}(\hat{M}_R) \) if

\[
\frac{1}{\pi^2} \left[ \lambda (C_M - \rho_c C_M) \lambda C_M^2 (1 - \rho^2) + \frac{\lambda C_M^2 (1 - \rho^2) \text{MSE}(\hat{M}_R)}{M_y^2} + \lambda^2 \theta_1 \right] > 0.
\]

**Condition (iii)**

By (2.7) and (3.5), \( \text{MSE}(\hat{M}^G_{\hat{Y}})_\text{min} < \text{MSE}(\hat{M}_{EX}) \) if

\[
\frac{1}{\pi^2} \left[ \lambda (C_M - \rho_c C_M) \lambda C_M^2 (1 - \rho^2) + \frac{M_y^2 \text{MSE}(\hat{M}_{EX})}{M_y^2} + \lambda^2 \theta_1 \right] > 0.
\]
Condition (iv)
By (2.9) and (3.5), \( \text{MSE}(\hat{M}_{GPP})_{\text{min}} < \text{MSE}(\hat{M}_{D})_{\text{min}} \) if
\[
\frac{1}{\sigma^2} \left[ \{\lambda C_{My}^2(1-\rho_c^2)\}^2 + \lambda^2 \theta_1 \right] > 0.
\]

Condition (v)
By (2.16) and (3.5), \( \text{MSE}(\hat{M}_{GPP})_{\text{min}} < \text{MSE}(\hat{M}_{D1})_{\text{min}} \) if
\[
\frac{1}{\sigma^2} \left[ \lambda^2 \theta_1 \right] > 0.
\]

Condition (vi)
By (2.17) and (3.5), \( \text{MSE}(\hat{M}_{GPP})_{\text{min}} < \text{MSE}(\hat{M}_{D2})_{\text{min}} \) if
\[
\frac{1}{\sigma^2} \left[ \lambda^2 C_{My}^2 C_{Mx}^2 (1-\rho_c^2)(1-\lambda C_{My}^2 C_{Mx}) + \frac{1}{4} \lambda^2 C_{Mx}^2 (1-\lambda C_{Mx}) \right] > 0,
\]
where \( \theta_3 = 1 - \lambda C_{Mx}^2 + \lambda C_{My} (1-\rho_c^2) \).

Condition (vii)
By (2.18) and (3.5), \( \text{MSE}(\hat{M}_{GPP})_{\text{min}} < \text{MSE}(\hat{M}_{D3})_{\text{min}} \) if
\[
\frac{1}{\sigma^2} \left[ \left( \frac{1}{4} \lambda^2 C_{Mx}^2 \right) \left( C_{My}^2 (1-\rho_c^2) + \frac{3}{10} C_{Mx}^2 \right) \right] > 0.
\]

5. Numerical study in simple random sampling
In this section, we consider seven natural populations to perform a numerical comparison of different estimators.

Population I: Source: [PDS [6], Pages 114-116]
Let \( y \) be the number of teaching staff and \( x \) be the number of students in 4 different types of schools under 36 districts in Punjab province of Pakistan.

Population II: Source: [Singh [13]]
Let \( y \) be the number of fish caught in the year 1995 and \( x \) be the number of fish caught by the marine recreational fishermen in the previous year 1994 in USA.

Population III: Source: [Singh [13]]
Let \( y \) be the number of fish caught in the year 1995 and \( x \) be the number of fish caught by the marine recreational fishermen in the previous year 1993 in USA.

Population IV: Source: [Aladag and Cingi [3]]
Let \( y \) be the number of teachers and \( x \) be the number of students in elementary schools for 340 medium-developed districts in Turkey in 2007.

Population V: Source: [Chen et al. [4]; Al and Cingi [2]]
Let \( y \) be the entire height of conifer trees in feet and \( x \) be the diameter of conifer trees in centimeters at breast height.

Population VI: Source: [Aczel and Sounderpandian [1]]
Let \( y \) be the U.S. exports to Singapore in billions of Singapore dollars and \( x \) be the money supply figures in billions of Singapore dollars.

Population VII: Source: [MFA [9]]
Let \( y \) be the district-wise tomato production in tons in Pakistan in the year 2003 and \( x \) be the district-wise tomato production in tons in Pakistan in the year 2002.

The summary data of Populations (I–VII) are given in Table 1. We use the following expression to obtain the percent relative efficiency (PRE) of various estimators relative to the sample median i.e.
\[
\text{PRE} = \frac{\text{MSE}(\hat{M}_y)}{\text{MSE}(\cdot) \text{ or } \text{MSE}(\cdot)_{\text{min}}} \times 100.
\]

The Bias, \( \text{MSE} \) and \( \text{PRE} \) results are given in Tables 2-4 respectively. Based on the results in Tables 2-4, it is observed that the proposed estimator \( \hat{M}_{GPP} \) outperforms other competing estimators. Also in Table 2, the absolute bias of \( \hat{M}_{GPP} \) is smaller in most
cases. The ratio estimator \( \hat{M}_R \) and exponential estimator \( \hat{M}_{EX} \) show poorest PREs probably because of weaker correlation between the study variable and the auxiliary variable. The performance of the proposed estimator \( \hat{M}_G \) is not affected by this weak correlation.

### Table 1. Summary statistics for seven populations.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Pop. I</th>
<th>Pop. II</th>
<th>Pop. III</th>
<th>Pop. IV</th>
<th>Pop. V</th>
<th>Pop. VI</th>
<th>Pop. VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>144</td>
<td>69</td>
<td>69</td>
<td>340</td>
<td>396</td>
<td>67</td>
<td>97</td>
</tr>
<tr>
<td>( n )</td>
<td>10</td>
<td>17</td>
<td>17</td>
<td>150</td>
<td>65</td>
<td>23</td>
<td>46</td>
</tr>
<tr>
<td>( M_y )</td>
<td>2023</td>
<td>2068</td>
<td>2068</td>
<td>178</td>
<td>30</td>
<td>4.8</td>
<td>1242</td>
</tr>
<tr>
<td>( M_x )</td>
<td>64659</td>
<td>2011</td>
<td>2307</td>
<td>3526</td>
<td>14.6</td>
<td>7.0</td>
<td>1233</td>
</tr>
<tr>
<td>( f_y(M_y) )</td>
<td>0.00024</td>
<td>0.00014</td>
<td>0.00014</td>
<td>0.00182</td>
<td>0.01178</td>
<td>0.07630</td>
<td>0.00021</td>
</tr>
<tr>
<td>( f_x(M_x) )</td>
<td>0.00001</td>
<td>0.00014</td>
<td>0.00014</td>
<td>0.00008</td>
<td>0.02194</td>
<td>0.05260</td>
<td>0.00022</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>0.8611</td>
<td>0.1505</td>
<td>0.3136</td>
<td>0.92</td>
<td>0.84</td>
<td>0.6624</td>
<td>0.2096</td>
</tr>
</tbody>
</table>

### Table 2. Bias of different estimators.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Pop. I</th>
<th>Pop. II</th>
<th>Pop. III</th>
<th>Pop. IV</th>
<th>Pop. V</th>
<th>Pop. VI</th>
<th>Pop. VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{M}_R )</td>
<td>-16.522</td>
<td>246.828</td>
<td>171.241</td>
<td>0.414</td>
<td>0.224</td>
<td>0.084</td>
<td>37.718</td>
</tr>
<tr>
<td>( \hat{M}_{EX} )</td>
<td>-22.332</td>
<td>87.271</td>
<td>53.769</td>
<td>-0.053</td>
<td>-0.005</td>
<td>0.011</td>
<td>12.829</td>
</tr>
<tr>
<td>( \hat{M}_{D1} )</td>
<td>-50.326</td>
<td>-236.651</td>
<td>-219.863</td>
<td>-0.242</td>
<td>-0.226</td>
<td>-0.139</td>
<td>-47.952</td>
</tr>
<tr>
<td>( \hat{M}_{D2} )</td>
<td>-50.253</td>
<td>-232.350</td>
<td>-216.626</td>
<td>-0.242</td>
<td>-0.226</td>
<td>-0.139</td>
<td>-47.878</td>
</tr>
<tr>
<td>( \hat{M}_{D3} )</td>
<td>-49.531</td>
<td>-227.820</td>
<td>-212.652</td>
<td>-0.241</td>
<td>-0.223</td>
<td>-0.137</td>
<td>-47.459</td>
</tr>
<tr>
<td>( \hat{M}_{G})</td>
<td>-45.999</td>
<td>-149.613</td>
<td>-185.758</td>
<td>-0.233</td>
<td>-0.211</td>
<td>-0.129</td>
<td>-45.640</td>
</tr>
</tbody>
</table>

### 6. Some median estimator in stratified random sampling

Stratified random sampling is commonly used when population is heterogeneous. Recently Aladag and Cingi [3] suggested some median estimators in stratified random sampling. We give below some notations and some of the existing median estimators in stratified sampling.

Consider a finite population \( U = (1, 2, \ldots, N) \) of \( N \) identifiable units divided into \( L \) strata with the \( h \)th stratum \( \{h = 1, 2, \ldots, L\} \) having \( N_h \) units such that

\[
\sum_{h=1}^{L} N_h = N.
\]

Let \( y_{hi} \) and \( x_{hi} \) be the values of the study variable \( Y_h \) and the auxiliary variable \( X_h \) respectively for the \( i \)th \((i = 1, 2, \ldots, N)\) population element of the \( h \)th
Table 3. MSE values of different estimators.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Pop. I</th>
<th>Pop. II</th>
<th>Pop. III</th>
<th>Pop. IV</th>
<th>Pop. V</th>
<th>Pop. VI</th>
<th>Pop. VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{M}_y$</td>
<td>403886.96</td>
<td>565443.57</td>
<td>565443.57</td>
<td>281.18</td>
<td>23.17</td>
<td>1.23</td>
<td>64795.08</td>
</tr>
<tr>
<td>$\hat{M}_R$</td>
<td>109312.95</td>
<td>988372.76</td>
<td>746752.56</td>
<td>57.87</td>
<td>8.43</td>
<td>0.82</td>
<td>98581.89</td>
</tr>
<tr>
<td>$\hat{M}_{EX}$</td>
<td>199668.00</td>
<td>627420.21</td>
<td>524362.05</td>
<td>76.80</td>
<td>8.75</td>
<td>0.72</td>
<td>66712.60</td>
</tr>
<tr>
<td>$\hat{M}_D$</td>
<td>104407.52</td>
<td>552636.13</td>
<td>508766.02</td>
<td>43.19</td>
<td>6.82</td>
<td>0.69</td>
<td>61948.49</td>
</tr>
<tr>
<td>$\hat{M}_{D1}$</td>
<td>101810.17</td>
<td>489395.24</td>
<td>454675.78</td>
<td>43.13</td>
<td>6.77</td>
<td>0.67</td>
<td>59556.73</td>
</tr>
<tr>
<td>$\hat{M}_{D2}$</td>
<td>101661.15</td>
<td>480458.29</td>
<td>447982.61</td>
<td>43.13</td>
<td>6.76</td>
<td>0.67</td>
<td>59463.97</td>
</tr>
<tr>
<td>$\hat{M}_{D3}$</td>
<td>100200.79</td>
<td>471131.76</td>
<td>439763.44</td>
<td>42.94</td>
<td>6.70</td>
<td>0.66</td>
<td>58943.58</td>
</tr>
<tr>
<td>$\hat{M}_{GP}$</td>
<td>93055.81</td>
<td>402459.28</td>
<td>384146.79</td>
<td>41.78</td>
<td>6.34</td>
<td>0.62</td>
<td>56684.80</td>
</tr>
</tbody>
</table>

Table 4. PRE of different estimators.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Pop. I</th>
<th>Pop. II</th>
<th>Pop. III</th>
<th>Pop. IV</th>
<th>Pop. V</th>
<th>Pop. VI</th>
<th>Pop. VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{M}_y$</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}_R$</td>
<td>369.48</td>
<td>57.21</td>
<td>75.72</td>
<td>485.91</td>
<td>274.79</td>
<td>148.89</td>
<td>65.73</td>
</tr>
<tr>
<td>$\hat{M}_{EX}$</td>
<td>202.28</td>
<td>90.12</td>
<td>107.84</td>
<td>366.10</td>
<td>264.86</td>
<td>169.93</td>
<td>97.13</td>
</tr>
<tr>
<td>$\hat{M}_D$</td>
<td>386.84</td>
<td>102.32</td>
<td>111.14</td>
<td>651.04</td>
<td>339.67</td>
<td>178.81</td>
<td>104.59</td>
</tr>
<tr>
<td>$\hat{M}_{D1}$</td>
<td>396.61</td>
<td>115.39</td>
<td>124.36</td>
<td>651.93</td>
<td>342.25</td>
<td>183.50</td>
<td>108.93</td>
</tr>
<tr>
<td>$\hat{M}_{D2}$</td>
<td>397.29</td>
<td>117.69</td>
<td>126.22</td>
<td>651.94</td>
<td>342.33</td>
<td>183.79</td>
<td>109.96</td>
</tr>
<tr>
<td>$\hat{M}_{D3}$</td>
<td>403.08</td>
<td>120.03</td>
<td>128.58</td>
<td>654.51</td>
<td>345.65</td>
<td>186.22</td>
<td>110.29</td>
</tr>
<tr>
<td>$\hat{M}_{GP}$</td>
<td>434.03</td>
<td>140.47</td>
<td>147.19</td>
<td>676.86</td>
<td>365.54</td>
<td>198.56</td>
<td>114.30</td>
</tr>
</tbody>
</table>

stratum. Let $\hat{M}_{yh}$ and $\hat{M}_{xh}$ be the sample medians respectively corresponding to population medians $M_{yh}$ and $M_{xh}$ in the hth stratum. Let $\hat{M}_{yst} = \sum_{h=1}^{L} W_h \hat{M}_{yh}$ and $\hat{M}_{xst} = \sum_{h=1}^{L} W_h \hat{M}_{xh}$ be weighted sample medians respectively corresponding to population medians $M_y = M_{yst} = \sum_{h=1}^{L} W_h M_{yh}$ and $M_x = M_{xst} = \sum_{h=1}^{L} W_h M_{xh}$, where $W_h = N_h / N$ is the known stratum weight. Let the correlation coefficient between $(\hat{M}_{yh}, \hat{M}_{xh})$ be $\rho(\hat{M}_{yh}, \hat{M}_{xh}) = \rho_{ch} = 4P(1h(y_h, x_h)) - 1$, where $P(1h(y_h, x_h)) = P(Y_h \leq M_{yh} \cap X_h \leq M_{xh})$. It is assumed that the distribution of $(Y_h, X_h)$ is a continuous distribution with marginal densities $f_{yh}(y_h)$ and $f_{xh}(x_h)$ for $Y_h$ and $X_h$ respectively. It is further assumed that $f_{yh}(M_{yh})$ and $f_{xh}(M_{xh})$ are positive.
To obtain expressions for the biases and $MSE$s of different estimators, we use the following error terms. Let $e_{0h} = (M_{yh} - M_{yh})/M_{yh}$ and $e_{1h} = (M_{zh} - M_{zh})/M_{zh}$ such that $E(e_{0h}) = E(e_{1h}) = 0$. To first degree of approximation, we have $E(e_{0h}^2) = \lambda_h C_{M_{yh}}^2$, $E(e_{1h}^2) = \lambda_h C_{M_{yh}}^2$, $E(e_{0h} e_{1h}) = \lambda_h C_{M_{yh}}$, where $C_{M_{yh}} = 1/[M_{yh}]f_{yh}(M_{yh})$, $C_{M_{zh}} = 1/[M_{zh}]f_{zh}(M_{zh})$, $C_{M_{yzh}} = \rho_{zh}C_{M_{yh}}C_{M_{zh}}$ and $\lambda_h = \frac{1}{2} \left( \frac{1}{\pi h} - \frac{1}{\pi h} \right)$.

The variance of the usual sample median $(\hat{M}_{yst})$ as an estimator of the population median $M_y$, is given by

$$
\text{Var}(\hat{M}_{yst}) = \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 C_{M_{yh}}^2 = MSE(\hat{M}_{yst}).
$$

The traditional ratio estimator in stratified sampling, is given by

$$
\hat{M}_{R} = \sum_{h=1}^{L} W_h M_{yh} \left( \frac{M_{zh}}{M_{zh}} \right),
$$

where $M_{yh}$ is known for all strata.

The bias and $MSE$ of $\hat{M}_{R}$, are given by

$$
\text{Bias}(\hat{M}_{R}) \approx \sum_{h=1}^{L} W_h \lambda_h M_{yh} \left( C_{M_{zh}}^2 - C_{M_{yzh}} \right),
$$

and

$$
MSE(\hat{M}_{R}) \approx \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 \left( C_{M_{yh}}^2 + C_{M_{yzh}}^2 - 2C_{M_{yzh}} \right).
$$

The exponential ratio type estimator in stratified sampling, is given by

$$
\hat{M}_{E_{xy}} = \sum_{h=1}^{L} W_h^2 M_{yh} \exp \left( \frac{M_{zh} - \hat{M}_{zh}}{M_{zh} + \hat{M}_{zh}} \right).
$$

The bias and $MSE$ of $\hat{M}_{E_{xy}}$, are given by

$$
\text{Bias}(\hat{M}_{E_{xy}}) \approx \sum_{h=1}^{L} W_h \lambda_h M_{yh} \left\{ \frac{3}{8} C_{M_{zh}}^2 - \frac{1}{2} C_{M_{yzh}} \right\},
$$

and

$$
MSE(\hat{M}_{E_{xy}}) \approx \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 \left\{ C_{M_{yh}}^2 + \frac{1}{4} C_{M_{yzh}}^2 - C_{M_{yzh}} \right\}.
$$

The unbiased difference estimator $\hat{M}_{D_{ch}}$, is given by

$$
\hat{M}_{D_{ch}} = \sum_{h=1}^{L} W_h \left[ \hat{M}_{yh} + d_h \left( M_{zh} - \hat{M}_{zh} \right) \right],
$$

where $d_h$ is an unknown constant.

The minimum $MSE$ of $\hat{M}_{D_{ch}}$, at optimum values of $d_h$, i.e. $d_h(\text{opt}) = \frac{M_{yh} \rho_{ch} C_{M_{yh}}}{M_{zh} C_{M_{zh}}}$, is given by

$$
MSE(\hat{M}_{D_{ch}})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 C_{M_{yh}}^2 \left( 1 - \rho_{ch}^2 \right).
$$

The minimum $MSE$ of $\hat{M}_{D_{ch}}$ is always smaller than the sample median estimator $(\hat{M}_{yst})$, ratio estimator $(\hat{M}_{R})$ and exponential type estimator $(\hat{M}_{E_{xy}})$. 


Some more difference type estimators \( \hat{M}_{DI,i} \) \((i = 1, 2, 3)\) in stratified random sampling, are given by:
\[
\hat{M}_{D1s} = \sum_{h=1}^{L} W_h \left[ d_{1h} \hat{M}_{yh} + d_{2h} \left( M_{xh} - \hat{M}_{xh} \right) \right],
\]
\[
\hat{M}_{D2s} = \sum_{h=1}^{L} W_h \left[ d_{3h} \hat{M}_{yh} + d_{4h} (M_{xh} - \hat{M}_{xh}) \right] \left( \frac{M_{xh}}{\hat{M}_{xh}} \right),
\]
\[
\hat{M}_{D3s} = \sum_{h=1}^{L} W_h \left[ d_{5h} \hat{M}_{yh} + d_{6h} (M_{xh} - \hat{M}_{xh}) \right] \exp \left( \frac{M_{xh} - \hat{M}_{xh}}{\hat{M}_{xh} + M_{xh}} \right),
\]
where \(d_{ih}(i = 1, 2, \ldots, 6)\) are unknown constants whose values are to be determined. The biases and minimum \(MSEs\) of \(\hat{M}_{DI,i} \) \((i = 1, 2, 3)\), are given by
\[
Bias(\hat{M}_{D1s}) \approx \sum_{h=1}^{L} W_h \hat{M}_{yh} (d_{1h} - 1),
\]
\[
Bias(\hat{M}_{D2s}) \approx \sum_{h=1}^{L} W_h \left[ (d_{2h} - 1) \hat{M}_{yh} + d_{3h} \lambda_h M_{yh} \left\{ C_{Mxh}^2 - C_{Myhx} \right\} + d_{4h} M_{xh} \lambda_h C_{Mxh}^2 \right],
\]
\[
Bias(\hat{M}_{D3s}) \approx \sum_{h=1}^{L} W_h \left[ (d_{5h} - 1) \hat{M}_{yh} + d_{6h} \lambda_h M_{yh} \left\{ \frac{3}{8} C_{Mxh}^2 - \frac{1}{2} C_{Myhx} \right\} + \frac{1}{2} d_{6h} M_{xh} \lambda_h C_{Mxh}^2 \right]
\]
\[
MSE(\hat{M}_{D1s})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 \hat{M}_{yh}^2 \lambda_h \left( \frac{C_{Myhx}(1 - \rho_{ch}^2)}{1 + \lambda_h C_{Mxh}^2 (1 - \rho_{ch}^2)} \right),
\]
\[
MSE(\hat{M}_{D2s})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 M_{yh}^2 \left[ 1 - \lambda_h C_{Mxh}^2 \right] - \frac{(1 - \lambda_h C_{Mxh}^2)^2}{(1 - \lambda_h C_{Mxh}^2) + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)},
\]
or
\[
MSE(\hat{M}_{D2s})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 M_{yh}^2 \left[ (1 - \lambda_h C_{Mxh}^2) \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2) \right] \left( 1 - \lambda_h C_{Mxh}^2 \right) + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2),
\]
\[
MSE(\hat{M}_{D3s})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 M_{yh}^2 \left[ 1 - \frac{1}{4} \lambda_h C_{Mxh}^2 \right] - \frac{(1 - \lambda_h C_{Mxh}^2)^2}{1 + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)},
\]
or
\[
MSE(\hat{M}_{D3s})_{\text{min}} \approx \sum_{h=1}^{L} W_h^2 M_{yh}^2 \left[ \frac{\lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)}{1 + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)} - \frac{1}{8} \lambda_h C_{Mxh}^2 C_{Myhx}^2 \right] + \frac{1}{8} \lambda_h C_{Mxh}^2 C_{Myhx}^2 (1 - \rho_{ch}^2).
\]
The optimum values of \(d_{ih}(i = 1, 2, \ldots, 6)\), are given by:
\[
d_{1h(\text{opt})} = \frac{1}{1 + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)}, \quad d_{2h(\text{opt})} = \frac{\hat{M}_{yh}}{M_{ych}^2} \left( \frac{\rho_{ch} C_{Myhx}/C_{Mxh}}{1 + \lambda_h C_{Myhx}^2 (1 - \rho_{ch}^2)} \right),
\]
By (6.1) and (7.4),

\[ d_{h(\text{opt})} = \frac{1 - \lambda_h C^2_{Myh}}{1 - \lambda_h C^2_{Myh} + \lambda_h C_{MMyh}(1 - r^2_{ch})}, \quad d_{4h(\text{opt})} = \frac{M_{yh}}{M_{xh}} \left[ 1 + d_{3h(\text{opt})} \left( \frac{\rho_{ch} C_{MMyh}}{C_{Mxh}} - 2 \right) \right]. \]

\[ d_{5h(\text{opt})} = \frac{1 - \lambda_h C^2_{Myh}}{1 - \lambda_h C^2_{Myh} + \lambda_h C_{MMyh}(1 - r^2_{ch})}, \quad d_{6h(\text{opt})} = \frac{M_{yh}}{M_{xh}} \left[ \frac{1}{2} + d_{5h(\text{opt})} \left( \frac{\rho_{ch} C_{MMyh}}{C_{Mxh}} - 1 \right) \right]. \]

7. Proposed median estimator in stratified random sampling

Motivated by Singh and Solanki [14], Haji et al. [7], Sharma and Singh [12], Aladag and Cingi [3] and Solanki and Singh [15], we propose the following general class of difference type estimators for \( M_y \) in stratified random sampling:

\[ \hat{M}^2_{MPS} = \sum_{h=1}^{L} W_h \left[ m_{1h} \hat{M}_{yh} + m_{2h}(M_{xh} - \hat{M}_{xh}) \right] \times \frac{\left( a_1 M_{xh} + b_1 \right)^{\alpha_{1h}} \exp \left( \frac{\alpha_{2h} \hat{M}_{yh} (M_{xh} - \hat{M}_{xh})}{a_1 (\gamma_h - 1) M_{xh} + \hat{M}_{xh} + 2b_1} \right)}{\lambda C^{\alpha_2}_{Myh}(1 - r^2_{ch})} \]

where \( a_1 \) and \( b_1 \) are the known population parameters.; \( m_{1h} \) and \( m_{2h} \) are unknown constants whose values are to be determined and \( \alpha_{1h}, \alpha_{2h}, \gamma_h \) are scalar quantities which can take different values.

Note: By substituting different values of \( \alpha_{1h}, \alpha_{2}, \gamma, a, b \), we can generate many estimators.

Putting \( \alpha_{1h} = b_1 = 0, \alpha_{2h} = \gamma_h = a_h = 1 \) in Eq. (7.1), a new class of generalized difference type estimators in stratified sampling becomes:

\[ \hat{M}^2_{MP_{PS}} = \sum_{h=1}^{L} W_h \left[ m_{1h} \hat{M}_{yh} + m_{2h}(M_{xh} - \hat{M}_{xh}) \right] \left[ \exp \left( \frac{M_{xh}}{M_{xh} - 1} \right) \right]. \]

Solving Eq. (7.2), the bias and minimum MSE of \( \hat{M}^2_{MP_{PS}} \) at optimum values: \( m_{1h(\text{opt})} = \frac{1 - \lambda_h C^2_{MMyh}}{1 + \lambda_h C^2_{MMyh}(1 - r^2_{ch})} \) and \( m_{2h(\text{opt})} = \frac{M_{yh}}{M_{xh}} \left[ 1 + m_{1h(\text{opt})} \left( \frac{\rho_{ch} C_{MMyh}}{C_{Mxh}} - 2 \right) \right] \), are given by

\[ \text{Bias}(\hat{M}^2_{MP_{PS}}) \approx \sum_{h=1}^{L} W_h (m_{1h(\text{opt})} - 1) \hat{M}_{yh} + m_{2h} M_y \lambda_h \left( \frac{3}{2} C^2_{MMyh} - C_{MMyh} \right) + m_{2h} M_{yh} \lambda C^2_{MMyh} \] and

\[ \text{MSE}(\hat{M}^2_{MP_{PS}}) \approx \sum_{h=1}^{L} W^2_{h} \hat{M}^2_{yh} \left[ (1 - \lambda_h C^2_{MMyh}) - \frac{(1 - \frac{1}{2} \lambda_h C^2_{MMyh})^2}{1 + \lambda_h C^2_{MMyh}(1 - r^2_{ch})} \right] \]

or

\[ \text{MSE}(\hat{M}^2_{MP_{PS}}) \approx \sum_{h=1}^{L} W^2_{h} \hat{M}^2_{yh} \left[ \frac{\lambda_h C^2_{MMyh}(1 - r^2_{ch}) - \lambda_h^2 C^2_{MMyh} + \lambda_h^2 C^2_{MMyh} C^2_{MMyh}(1 - r^2_{ch})}{1 + \lambda_h C^2_{MMyh}(1 - r^2_{ch})} \right]. \]

8. Comparison of estimators in stratified random sampling

In this section, we compare the proposed class of generalized difference type estimators \( \hat{M}^2_{MP_{PS}} \) with other existing estimators.

**Condition (i)**

By (6.1) and (7.4), \( \text{MSE}(\hat{M}^2_{MP_{PS}})_{\text{min}} < \text{MSE}(\hat{M}_{y}) \) if

\[ \sum_{h=1}^{L} W^2_{h} \hat{M}^2_{yh} \left[ \lambda_h C^2_{MMyh} + \lambda^2_h \theta_{ch} \right] > 0, \]

where \( \theta_{ch} = C^2_{MMyh}(1 - r^2_{ch}) + C^2_{MMyh}(1 - r^2_{ch}) + \frac{1}{2} C^3_{MMyh} \) and
\[ \theta_{2h} = 1 + \lambda_h C_{M|yh}^2 (1 - \rho_{ch}^2). \]

**Condition (ii)**
By (6.4) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE} (\hat{M}_{Ds})\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ \lambda_h (C_{M|xh} - \rho_{ch} C_{M|yh})^2 \right] > 0.
\]
where \(\theta_{2h} = C_{M|yh}^2 (1 - \rho_{ch}^2) + \lambda_h^2 (\theta_{1h}^* + \theta_{4h}) \text{ and } \theta_{4h} = C_{M|yh}^2 (1 - \rho_{ch}^2) (C_{M|yh}^2 + C_{M|xh}^2 - 2C_{M|yxh}). \)

**Condition (iii)**
By (6.7) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE}(\hat{M}_{EXs})\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ \lambda_h (\frac{1}{2} C_{M|xh} - \rho_{ch} C_{M|yh})^2 \right] > 0.
\]
where \(\theta_{4h} = C_{M|yh}^2 (1 - \rho_{ch}^2) (C_{M|yh}^2 + \frac{1}{4} C_{M|xh}^2 - C_{M|yxh}). \)

**Condition (iv)**
By (6.9) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE}(\hat{M}_{Ds})_{\text{min}}\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ \lambda_h^2 (C_{M|yh}^2) \right] > 0.
\]

**Condition (v)**
By (6.16) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE}(\hat{M}_{D_1s})_{\text{min}}\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ \lambda_h^2 \theta_{1h}^* \right] > 0.
\]

**Condition (vi)**
By (6.17) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE}(\hat{M}_{D_2s})_{\text{min}}\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ C_{M|yxh}^2 \lambda_h C_{M|xh}^2 (1 - \rho_{ch}^2) (1 - \frac{1}{2} \lambda_h C_{M|xh}) + \theta_{4h} \right] > 0.
\]
where \(\theta_{4h} = 1 - \lambda_h C_{M|xh}^2 + \lambda_h C_{M|yh}^2 (1 - \rho_{ch}^2) \text{ and } \theta_{4h} = \frac{1}{4} C_{M|xh}^2 (1 - \lambda_h C_{M|xh}). \)

**Condition (vii)**
By (6.18) and (7.4), \(\text{MSE}(\hat{M}_{PPs})_{\text{min}} < \text{MSE}(\hat{M}_{D_3s})_{\text{min}}\) if
\[
\sum_{h=1}^{L} W_h^2 M_{2h} \frac{1}{\varphi_{2h}} \left[ \frac{1}{2} \lambda_h^2 C_{M|xh}^2 \right] \left[ C_{M|yh}^2 (1 - \rho_{ch}^2) + \frac{1}{4} C_{M|xh}^2 \right] \right] > 0.
\]

### 9. A numerical study in stratified random sampling

In this section, we consider the following two populations to perform a numerical comparison of different estimators in stratified random sampling.

**Population I:** [source: PDS [6]]
Let \(y\) be the number of teaching staff as the study variable and \(x\) be the number of students as the auxiliary variable in 4 different types of schools under 36 districts in Punjab (Pakistan). Equal allocation was used to obtain the sample size in each stratum.

**Population II:** [source: [Aladag and Cingi [3]]
Let \(y\) be the number of teachers as the study variable and \(x\) be the number of students as the auxiliary variable in both primary and secondary schools for 923 districts in 6 regions (as 1—Marmara, 2—Aegean, 3—Mediterranean, 4—Central Anatolia, 5—Black Sea, 6—East and South Anatolia) in Turkey in 2007. The Neyman allocation was used for allocating the samples to different strata. The descriptive statistics are given in Tables 5 and 6. We use the following expression to obtain the percent relative efficiency (PRE) of various estimators with respect to the sample median i.e.
\[
PRE = \frac{\text{MSE} (\hat{M}_{gst})}{\text{MSE}(\cdot) or \text{MSE}(\cdot)_{\text{min}}} \times 100.
\]
Table 5. Data statistics for population I under stratified random sampling.

<table>
<thead>
<tr>
<th></th>
<th>N = 144</th>
<th>N₁ = 36</th>
<th>N₂ = 36</th>
<th>N₃ = 36</th>
<th>N₄ = 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>20</td>
<td>n₁ = 5</td>
<td>n₂ = 5</td>
<td>n₃ = 5</td>
<td>n₄ = 5</td>
</tr>
<tr>
<td>Mₓ₁</td>
<td>38</td>
<td>Mₓ₂ = 3056</td>
<td>Mₓ₃ = 2033</td>
<td>Mₓ₄ = 2382</td>
<td>Mₓ₅ = 1480</td>
</tr>
<tr>
<td>Mₓ₂</td>
<td>127289</td>
<td>Mₓ₃ = 5459</td>
<td>Mₓ₄ = 71615</td>
<td>f₁ₓ₁(Mₓ₃) = 0.007056</td>
<td>f₁ₓ₁(Mₓ₄) = 0.003202</td>
</tr>
<tr>
<td>f₁ₓ₁(Mₓ₁) = 0.0004219</td>
<td>f₁ₓ₁(Mₓ₂) = 0.0003012</td>
<td>f₁ₓ₁(Mₓ₃) = 0.0001641</td>
<td>f₁ₓ₁(Mₓ₄) = 0.00007827</td>
<td>f₁ₓ₁(Mₓ₅) = 0.0000141</td>
<td></td>
</tr>
<tr>
<td>f₁ₓ₁(Mₓ₁) = 0.0001026</td>
<td>ρ₁ = 0.7776</td>
<td>ρ₂ = 0.8888</td>
<td>ρ₃ = 0.8888</td>
<td>ρ₄ = 0.8888</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Data statistics for population II under stratified random sampling.

<table>
<thead>
<tr>
<th></th>
<th>N = 923</th>
<th>N₁ = 91</th>
<th>N₂ = 129</th>
<th>N₃ = 204</th>
<th>N₄ = 145</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>184</td>
<td>n₁ = 177</td>
<td>n₂ = 18</td>
<td>n₃ = 26</td>
<td>n₄ = 34</td>
</tr>
<tr>
<td>Mₓ₁</td>
<td>41</td>
<td>Mₓ₂ = 24</td>
<td>Mₓ₃ = 54</td>
<td>Mₓ₄ = 44</td>
<td>Mₓ₅ = 101</td>
</tr>
<tr>
<td>Mₓ₂</td>
<td>1265</td>
<td>Mₓ₃ = 1139</td>
<td>Mₓ₄ = 614</td>
<td>Mₓ₅ = 763</td>
<td>Mₓ₆ = 533</td>
</tr>
<tr>
<td>f₁ₓ₁(Mₓ₃) = 0.003160</td>
<td>f₁ₓ₁(Mₓ₄) = 0.003180</td>
<td>f₁ₓ₁(Mₓ₅) = 0.011510</td>
<td>f₁ₓ₁(Mₓ₆) = 0.000299</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f₁ₓ₁(Mₓ₃) = 0.005120</td>
<td>f₁ₓ₁(Mₓ₄) = 0.000249</td>
<td>f₁ₓ₁(Mₓ₅) = 0.000190</td>
<td>f₁ₓ₁(Mₓ₆) = 0.000240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f₁ₓ₁(Mₓ₃) = 0.004420</td>
<td>f₁ₓ₁(Mₓ₄) = 0.000523</td>
<td>f₁ₓ₁(Mₓ₅) = 0.000087</td>
<td>ρ₁ = 0.84</td>
<td>ρ₄ = 0.96</td>
<td></td>
</tr>
<tr>
<td>ρ₁ = 0.84</td>
<td>ρ₂ = 0.88</td>
<td>ρ₃ = 0.88</td>
<td>ρ₅ = 0.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results are given in Tables 7 and 8. From Table 7, we observed that the absolute
bias for the proposed estimators $\hat{M}^{G}_{PPs}$ is the smallest in most cases. In Table 8, $\hat{M}^{G}_{PPs}$ has the highest percent relative efficiency as compared to all other estimators.

Table 7. Bias of different estimators under stratified random sampling.

<table>
<thead>
<tr>
<th></th>
<th>Estimator</th>
<th>Population I</th>
<th>Population II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{M}_{Rs}$</td>
<td>16.977</td>
<td>-35.781</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}_{EXs}$</td>
<td>-7.079</td>
<td>-20.569</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}_{D1s}$</td>
<td>-25.427</td>
<td>-15.952</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}_{D2s}$</td>
<td>-24.777</td>
<td>-9.186</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}_{D3s}$</td>
<td>-24.513</td>
<td>-13.930</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}^{G}_{PPs}$</td>
<td>-19.541</td>
<td>-6.001</td>
<td></td>
</tr>
</tbody>
</table>
Table 8. MSE and PRE of different estimators with respect to $\hat{M}_{gs}$.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population I</th>
<th>Population II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>PRE</td>
</tr>
<tr>
<td>$\hat{M}_{gs}$</td>
<td>71075.12</td>
<td>100.000</td>
</tr>
<tr>
<td>$\hat{M}_{Rs}$</td>
<td>16174.55</td>
<td>439.426</td>
</tr>
<tr>
<td>$\hat{M}_{EXs}$</td>
<td>25510.05</td>
<td>278.616</td>
</tr>
<tr>
<td>$\hat{M}_{Ds}$</td>
<td>14938.38</td>
<td>475.789</td>
</tr>
<tr>
<td>$\hat{M}_{D1s}$</td>
<td>14736.57</td>
<td>482.304</td>
</tr>
<tr>
<td>$\hat{M}_{D2s}$</td>
<td>14716.18</td>
<td>482.973</td>
</tr>
<tr>
<td>$\hat{M}_{D3s}$</td>
<td>14421.73</td>
<td>492.833</td>
</tr>
<tr>
<td>$\hat{M}_{GPPs}$</td>
<td>12597.10</td>
<td>564.218</td>
</tr>
</tbody>
</table>

10. Conclusion

We have proposed a generalized class of difference type estimators for finite population median in both simple and stratified random sampling. Some well-known estimators are particular members of the proposed classes of estimators. Numerical comparisons with other estimators show that the proposed new class of estimators $\hat{M}^{G}_{PPs}$ is more efficient both in simple and stratified random sampling.

Acknowledgments

The authors wish to thank the editor and the anonymous referees for their suggestions which led to improvement in the earlier version of the manuscript.

References