ALTERING DISTANCE AND COMMON
FIXED POINTS UNDER IMPLICIT
RELATIONS

Valeriu Popa* and Marcelina Mocanu†

Received 10 : 06 : 2009 : Accepted 29 : 09 : 2009

Abstract
We prove a common fixed point theorem for two pairs of occasionally weakly compatible mappings satisfying an implicit relation of a new type that involves an altering distance, so generalizing a theorem of Aliouche and Djoudi (Common fixed point theorems for mappings satisfying an implicit relation without decreasing assumption, Hacettepe J. Math. Stat. 36 (1), 11–18, 2007). As a consequence, we obtain a fixed point theorem for two pairs of mappings satisfying an implicit relation of integral type, providing a strong generalization to a known result from Kumar, Chugh and Kumar (Fixed point theorems for compatible mappings satisfying a contractive condition of integral type, Soochow J. Math. 33 (2), 181–185, 2007).

Keywords: Point of coincidence, Common fixed point, Occasionally weakly compatible mappings, Altering distance.


1. Introduction
Let $S$ and $T$ be self-mappings of a metric space $(X,d)$. Jungck [6] defined $S$ and $T$ to be compatible if $\lim_{n \to \infty} d(STx_n, TSx_n) = 0$ whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Tx_n = t$ for some $t \in X$.

1.1. Definition. Let $X$ be a non-empty set and $S,T$ self-mappings of $X$. A point $x \in X$ is called a coincidence point of $S$ and $T$ if $Sx = Tx$. A point $w \in X$ is said to be a point of coincidence of $S$ and $T$ if there exists $x \in X$ so that $w = Sx = Tx$. **

*Department of Mathematics and Informatics, Faculty of Sciences, University of Bacău, Spiru Haret 8, 600114 Bacău, Romania. E-mail: (V. Popa) vpopa@ub.ro (M. Mocanu) mmocanu@ub.ro
†Corresponding Author.