σ-REGULAR MATRICES AND A σ-CORE THEOREM FOR DOUBLE SEQUENCES

Celal Çakan*, Bilal Altay* and Hüsamettin Coşkun*

Received 31:10:2008 : Accepted 19:01:2009

Abstract

The famous Knopp Core of a single sequence was extended to the P-core of a double sequence by R.F. Patterson. Recently, the MR-core and σ-core of real bounded double sequences have been introduced and some inequalities analogues to those for Knopp’s Core Theorem have been studied. The aim of this paper is to characterize a class of four-dimensional matrices, and so to obtain necessary and sufficient conditions for a new inequality related to the P- and σ-cores.

Keywords: Double sequences, Invariant means, Core theorems and matrix transformations.

2000 AMS Classification: 40 C 05, 40 J 05, 46 A 45.

1. Introduction

A double sequence $x = [x_{jk}]_{j,k=0}^\infty$ is said to be convergent to a number $l$ in the sense of Pringsheim, or to be P-convergent, if for every $\varepsilon > 0$ there exists $N \in \mathbb{N}$, the set of natural numbers, such that $|x_{jk} - l| < \varepsilon$ whenever $j, k > N$, [11]. In this case, we write $P\text{-lim } x = l$. In what follows, we will write $[x_{jk}]$ in place of $[x_{jk}]_{j,k=0}^\infty$.

A double sequence $x$ is said to be bounded if there exists a positive number $M$ such that $|x_{jk}| < M$ for all $j, k$, i.e.,

$$||x|| = \sup_{j,k} |x_{jk}| < \infty.$$  

We note that in contrast to the case for single sequences, a convergent double sequence need not be bounded. By $c_0^\infty$, we mean the space of all P-convergent and bounded double sequences.

*İnönü University, Faculty of Education, Malatya-44280, Turkey.
E-mail: (C. Çakan) ccakan@inonu.edu.tr (B. Altay) baltay@inonu.edu.tr (H. Coşkun) hcoskun@inonu.edu.tr