

SOME NOTES CONCERNING CHEEGER-GROMOLL METRICS

Filiz Ağca * † and A. A. Salimov ‡

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Abstract

The purpose of this paper is to introduce Cheeger-Gromoll type metric on the cotangent bundle of Riemannian manifold and investigate curvature properties and geodesics on the cotangent bundle with respect to the Cheeger-Gromoll metric.

Keywords: Cheeger-Gromoll metric, cotangent bundle, vertical and horizontal lift, curvature tensor, geodesics

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1. Introduction

In [4] Cheeger and Gromoll study complete manifolds of nonnegative curvature and suggest a construction of a new Riemannian metrics ${}^{CG}g$. Musso and Triccerri [9] were the first giving the explicit formula for this metric. In [12] the Levi-Civita connection of ${}^{CG}g$ and its Riemannian curvature tensor are calculated by Sekizawa. In [5] Gudmundsson and Kappos corrected the formulas for curvature of ${}^{CG}g$ in the tangent bundle given by Sekizawa [12]. In [11] Salimov and Kazimova investigated geodesics of the Cheeger-Gromoll metric on tangent bundle. The geometry of Cheeger-Gromoll metric is well known and intensively studied for the tangent bundle (see for example [1],[2],[7],[8],[10]). The similar metric in theoretical physics has been obtained by Tamm (Nobel Laureate in Physics for the year 1958, see [13]). The main purpose of this paper is to introduce Levi-Civita connection of Cheeger-Gromoll type metric on the cotangent bundle T^*M^n of Riemannian manifold M^n and investigate curvature properties and geodesics on T^*M^n with respect to the Levi-Civita connection of ${}^{CG}g$. Since the construction of lifts to the cotangent bundle is not similar to the definition of lifts to the tangent bundle, we have some differences for Cheeger-Gromoll metrics on cotangent bundles (see Theorem 3.2 and Theorem 3.5).

Let (M^n, g) be an n -dimensional Riemannian manifold, T^*M^n its cotangent bundle

*Department of Mathematics, Karadeniz Technical University, 61080 Trabzon, Turkey.
E-mail: (F. Ağca) filiz_math@hotmail.com.

†Corresponding Author.

‡Department of Mathematics, Ataturk University, 25240 Erzurum, Turkey.
E-mail: (A. A. Salimov) asalimov@hotmail.com, asalimov@atauni.edu.tr