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HALF-INVERSE SPECTRAL PROBLEM FOR DIFFERENTIAL PENCILS WITH INTERACTION-POINT AND EIGENVALUE-DEPENDENT BOUNDARY CONDITIONS

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Abstract

The inverse spectral problem of recovering for a quadratic pencil of Sturm-Liouville operators with the interaction point and the eigenvalue parameter linearly contained in the boundary conditions are studied. The uniqueness theorem for the solution of the inverse problem according to the Weyl function is proved and a constructive procedure for finding its solution is obtained.

Keywords: Inverse spectral problem; Quadratic pencil of Sturm-Liouville operators; Eigenvalue-dependent boundary conditions; Interaction point.

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1. Introduction

We consider the boundary value problem (BVP) $L = L(q(x), \alpha, h_j, H_j,\ j = 0, 1)$:

(1.1) $$ly := y'' + (\lambda^2 - q(x))y = 0, \quad x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right),$$

(1.2) $$U(y) := y'(0) - (h_1\lambda + h_0)y(0) = 0,$$

(1.3) $$V(y) := y'(\pi) + (H_1\lambda + H_0)y(\pi) = 0,$$

(1.4) $$I(y) := \left\{ \begin{array}{c} y\left(\frac{\pi}{2}+0\right) = y\left(\frac{\pi}{2}-0\right) = y\left(\frac{\pi}{2}\right), \\ y'\left(\frac{\pi}{2}+0\right) - y'\left(\frac{\pi}{2}-0\right) = 2\alpha\lambda y\left(\frac{\pi}{2}\right), \end{array} \right.$$

where the potential $q(x) \in L_1(0, \pi)$ is a complex-valued function, $\alpha, h_j, H_j \in \mathbb{C}$, $j = 0, 1$; $h_1H_1 = -1$ and $\alpha(h_1 + H_1) - 2 \neq \pm i(h_1 + H_1 + 2\alpha)$, λ is a spectral parameter.

Notice that, we can understand problem (1.1),(1.4) as one of the treatments of the equation

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