SOME $\triangle$-CONVERGENCE THEOREMS
IN CAT(0) SPACES

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Abstract
In this paper, we use an iteration process for approximating common fixed points of two nonexpansive mappings by $\triangle$- and strong convergence in CAT(0) spaces. This process is independent of and simpler than the Ishikawa type iteration process.

Keywords: Iteration Process, Nonexpansive mappings, Common fixed points, $\triangle$-convergence, CAT(0) space.

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1. Introduction and Preliminaries

The notion of $\triangle$-convergence in general metric spaces was introduced by Lim [14] in 1976. Kirk and Panyanak [11] specialized this concept to CAT(0) spaces and showed that many Banach space results involving weak convergence have precise analogs in this setting. Dhompongsa and Panyanak [7] continued to work in this direction. Their results involved Mann and Ishikawa iteration schemes involving one mapping. In this paper, we explore common fixed points of two nonexpansive mapping by an iteration scheme which is both independent and simpler than the Ishikawa type iteration scheme.

For the sake of completeness, let us recall some definitions and known results in the existing literature on this subject :

1.1. Definition. A metric space $X$ is called a CAT(0) space [10] if it is geodesically connected and every geodesic triangle in $X$ is at least as “thin” as its comparison triangle in Euclidean plane. For a vigorous discussion, see Bridson and Haefliger [1], Bruhat and Tits [2], and Burago-Burago-Ivanov [3]. The complex Hilbert ball with a hyperbolic metric is a CAT(0) space, see [9] and [15].

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