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BOUR'S THEOREM ON THE GAUSS MAP IN 3-EUCLIDEAN SPACE

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Abstract

In this paper some relations are established between the Laplace-Beltrami operator and the curvatures of helicoidal surfaces in 3-Euclidean space. In addition, Bour's theorem on the Gauss map, and some special examples are given.

Keywords: Rotational surface, Helicoidal surface, Gauss map, Laplace-Beltrami operator, Minimal harmonic surface.

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1. Introduction

Surface theory in three dimensional Euclidean space have been studied for a long time, and many examples of such surfaces have been discovered. Many very useful books have been written on the subject, such as [7, 8].

In classical surface geometry in 3-Euclidean space, it is well known that the right helicoid (resp. catenoid) is the only ruled (resp. rotational) surface which is minimal. Moreover, a pair of these two surfaces has interesting properties. That is, they are both members of a one parameter family of isometric minimal surfaces, and have the same Gauss map. This pair is a typical example for minimal surfaces. On the other hand, the pair consisting of the right helicoid and the catenoid has the following generalization.

1.1. Theorem. ***Bour's Theorem****. A generalized helicoid is isometric to a surface of revolution so that helices on the helicoid correspond to parallel circles on the surface of revolution* [2].

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